Probabilistic Evaluation of a Power System’s Reliability and Quality Measures

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Abstract—Reliability and performance quality measures computed so far are deterministic in nature. They represent one operating state (a snapshot of the system conditions) in which the required demand and generation and transmission capacities are known with 100% certainty. In this paper a general and coherent formulation is presented, which can be used to account for the randomness associated with the load level as well as the availability of generation and transmission capacities. The general probability formulation can be used to calculate various reliability indices and quality measures. The paper describes the new approach for computing probabilistic evaluation (expected value) of the reliability indices and performance quality measures and presents illustrative applications. The methodology used in this paper constitutes a new line of research in the probabilistic reliability evaluation of a system where the derived system-wide performance quality indices are capable of classifying and exhibitionistic areas of deficiencies, bottlenecks and redundancies in large-scale power grids.

Keywords—probabilistic reliability; evaluation; quality; power systems

I. INTRODUCTION

The competitive electricity market is subjected to an increasing amount of uncertainties such as the demand forecast uncertainty, electricity price volatility, reliability of the generation groups, economic growth uncertainties, and changing environmental and social impacts on the energy sector [1-2]. In addition to considerations of these uncertainties, planners also need to balance the technical requirement of the system and the requirements from investors who aim at profit maximization [3-5]. There is a general utilization of reliability models throughout the world as the advantages of probabilistic methods over the deterministic approaches of planning, designing, operating and maintaining electric utility systems have clearly been recognized [6]. As has happened with many power system disciplines, the prime interest in system security, adequacy, and reliability has gradually shifted from completing and refining the theoretical basis, through developing suitable computational tools for demonstrating the capability and practicality of the methodologies, to upgrading the computational tools to handle the large-scale nature of present power systems and, finally, to relate various security, quality and reliability indices to the practical concerns of utility engineers and executives regarding supply and/or transmission deficiencies as well as the risk associated with ignoring such deficiencies [7-10]. Methods for computing probabilistic contingency-based reliability and performance quality indices have previously been published in [11-17]. These methods are based on a combined contingency analysis and reliability evaluation scheme which integrates both the contingency effect and its probability of occurrence into one routine of analysis [18-19]. In the current work, similar analysis will be used to compute the expected values of different system reliability and performance quality indices. In this context, a “contingency scenario” or a system “demand level” are regarded, in a more general sense, as a “state”, which occurs with certain probability and represents a given demand value and an availability pattern of various capacities in the system. This paper shares the results of a recent major industry supported research and development study in which a novel framework was developed [2] for evaluating performance quality indices associated with power system generation demand balance. In this paper, the optimization technique based on three metaphors (dimensions) representing the relationship between available generation capacities and required demand levels, is presented. The novel formulation presented in this paper can accommodate the randomness associated with the load level and the availability of generation and transmission capacities.

II. PROBLEM FORMULATION

A. Power System Network Model

Let \( n_b \) be the number of buses in the power network, where \( n_b = n_L + n_G \), with \( n_L \) and \( n_G \) being the number of load and generator buses respectively. Also, in the network model used, \( n_T \) is the number of transmission branches (lines and transformers). In order to facilitate the subsequent formulation, it is assumed without loss of generality, that the load buses are numbered as 1, 2, ..., \( n_L \), followed by generator buses as \( n_L + 1, \ldots, n_L + n_G \), where \( n_L + n_G = n_B \). For example, the sample power system shown in Figure 1 has \( n_B = 4, n_G = 2, n_L = 2 \) and \( n_T = 5 \). Now, let \( A = (n_B \times n_T) \) be the bus incidence matrix representing the connectivity pattern between buses and lines. The entries of \( A \) are 0, 1 or -1. Therefore, an element \( A_{bt} = 1 \) if bus \( b \) is feeding a transmission branch \( t, A_{bt} = -1 \) if bus \( b \) is fed from a branch \( t, \) and \( A_{bt} = 0 \) otherwise. In the current analysis, the \( A \) matrix is partitioned row-wise into \( A_L \) and \( A_G \) associated respectively with load and generator buses. The rows of \( A \) (or columns of...
\( A^T \) represent groups of buses while the columns of \( A \) (or rows of \( A^T \)) represent groups of transmission links. We also note that for practical large-scale networks, the matrix \( A \) is extremely sparse.

introduced in the previous section, we may say that a positive amount of \((P_g - \sum_{t \in T_g} \bar{P}_t)\) of generation beyond bus \( g \) has been bottled (blocked from usage). We should note that such a definition applies to a specific scenario of system configuration (the \( A \)-matrix) and loading conditions. For example, in the above discussion, we assumed that the set \( T_g \) does not represent any of pre-defined contingency scenarios. That is, \( T_g \) represents the full transmission capacity at bus \( g \). In addition to the above definitions, we also define, using similar notation, the following vector for later use: \( \bar{P}_g \) - vector of generation site capacities, which represents the maximum future expanded generation capacity that could be available at the same generation site. The novel framework applied in this paper is based on the original work in [2], in which three dimensions were introduced to represent the relationship between certain system generation capacity and the demand. These tropes relate to the following demand fulfillment issues:

- Need of capacity for demand fulfillment
- Existence of capacity (availability for demand fulfillment)
- Ability of capacity to reach the demand

The first trope defines whether the capacity is needed, the second defines whether the capacity exists, and the last defines whether the capacity can reach (delivered to) the demand. The eight possible combinations associated with the 0/1 (Yes/No) values of the three tropes, are illustrated in Table I. Generation quality indices are defined in terms of the previously defined “1/0” states indicating the (Needed, Exists, Can-reach) true/false values associated with each quality metaphor. We shall use the symbol \( Q_{ijk} \) to indicate the generation quality index state. Also, in the following expressions, we shall use Min \( \{ x, y, .., z \} \) to indicate the minimum of \( x, y, .., z \). The notation \( <\infty> \) will be used to denote \( Max \{ 0, x \} \), which is the maximum of \( x \) and zero \((=x \text{ if } x>0, \text{ or } 0 \text{ otherwise})\).

Table I summarizes the considered quality indices, namely the Utilized Generation Capacity (Q111), Bottled Generation Capacity (Q110), Deficit Generation Capacity (Q100), Surplus Generation Capacity (Q010), Bottled Generation Capacity (Q011), Redundant Generation Capacity (Q010), Surplus Generation Capacity (Q001), and Saved Generation Capacity (Q000).

C. Linear Program Formulation

In the computational scheme of [2], the integrated system quality assessment is performed via solving a master linear programming problem in which a feasible power flow is established which minimizes the total system Load Not Served (LNS) subject to capacity limits and flow equations. The master
linear program, which utilizes the network bus incidence matrix \( A \), is formulated as:

\[
\text{Objective function} = \text{Minimize } f = \sum_{j=1}^{n} (-P_j)
\]
with respect to \( P_L \), \( P_G \), and \( P_T \), subject to:

\[
A \cdot P = \begin{bmatrix} P_L \\ P_G \\ P_T \end{bmatrix} = \begin{bmatrix} -P_L \\ P_G \\ -P_T \end{bmatrix} \]

An optimization software package (CPLEX) has been used to solve the Master Linear Program. The overall process of the evaluation of power systems reliability and quality measures is summarized in the flowchart in Figure 3.

\[ L_{LOLP}(m) = \lambda_i^{m} f_m \quad (4) \]
represents the system loss of load probability for any operating state \( m \) (load level, loss of generation and/or transmission capacities) in the power grid.

\[
\lambda_i^{(m)} = \begin{cases} 0 & \text{if } P_i^{(m)} \leq P_i^0 \\ 1 & \text{if } P_i^{(m)} > P_i^0 \end{cases} \quad (5)
\]
and \( P_i^0 \) denotes the scheduled (required) load at load bus \( i \). Also, in (2), \( M_i \) denotes the number of all possible states.

III. PROBABILISTIC EVALUATION

The reliability and performance quality indices computed so far are deterministic in nature, i.e., they represent one operating state in which the required demand and the generation and transmission capacities are known with 100% certainty.

A. Proposed Formulation

In real life, load variations occur randomly and contingencies cause some generation and/or transmission capacities to be lost (become unavailable). In other words, neither the load levels nor the generation or transmission capacities are known with absolute certainty. They are rather subject to random variations and consequently the calculated reliability and performance quality indices are all subject to random variations where only expected values of these indices can be evaluated. For example, the load variations, which are accounted for using the so-called “load-duration curves” can be used to calculate the expected value of the LNS, which is widely known as the Expected Load Not-Served (eLNS). On the other hand, the randomness in the generation and transmission capacity availability are accounted for using the so-called forced-outage rates (or availability rates) associated with various facilities. Consequently, the expected values of the performance quality indices \( Q_{eLNS}^{111}, Q_{eLNS}^{110}, Q_{eLNS}^{101}, \) etc. can be evaluated using the modeled randomness of the system load as well as the generation and transmission capacity availabilities as will be outlined in this section. Methods for computing probabilistic contingency-based reliability and performance quality indices have previously been published [7]. These methods are based on a combined contingency analysis and reliability evaluation scheme which integrates both the contingency effect and its probability of occurrence into one routine of analysis. In the present work, similar analysis will be used to compute the expected values of different system performance quality indices. In this context, a “contingency scenario” or a system “demand level” are regarded, in a more general sense, as a “state” which occurs with certain probability and represents a given demand value and availability pattern of various capacities in the system.

B. Simulation Results

Consider the sample power system in Figure 4. The system consists of two generators, namely G1 and G2, with a total capacity of 100MW and connected to two load buses, with a
total required load of 115MW, via five transmission lines. In this illustrative example, the availability of the two generating units are assumed as 0.9 and 0.8, respectively (forced outage rates of G1 and G2, respectively). Also, for simplicity, all transmission lines are assumed to have availability of 0.85. The system load is assumed to have three possible levels, namely 35MW, 75MW, and 115MW with probability of occurrence 0.5, 0.3 and 0.2 respectively.

The combined generation/transmission state probability table associated with the generation and transmission capacities of this system, along with the reliability and performance quality indices $LNS$, $Q_{g111}$ and $Q_{g110}$ are shown in Table II for the 35MW load level. The rows of Table II represent various operating states. The first three columns of Table II represent the well-known generation Capacity Outage Probability Table (COPT) [20]. The next five columns (T1 to T5) represent the COPT for the transmission facilities of this system. Column ($\sum P_g$) is the total generation available capacities for each operating state shown in the Table. The column ($\sum P_g$) represents the actual total generation as calculated from the solution of the Master Linear Program for various operating states. Similarly, the next three columns represent the associated values of the indices LNS and $Q_{g111}$ and $Q_{g110}$ respectively. When these values are multiplied by the operating state probability shown in the next column, they produce the expected values of these indices as outlined in the last three columns of the Table, namely $eLNS$ and $eQ_{g111}$ and $eQ_{g110}$. Using the generation state availabilities shown in Table II, the discrete probability density functions of various capacities and indices can be evaluated and displayed. These discrete density functions show the overall probabilities of occurrence associated with given capacities or indices. For example, the probability density function of the generation capacity (state probabilities) for this system are depicted in Figure 5.

Figure 6 shows the probability density of the Utilized Generation Capacity ($Q_{g111}$) at 75MW load, while Figure 7 shows the probability density of Bottled Generation Capacity ($Q_{g110}$) at 115MW load and the probability density of Surplus Generation Capacity ($Q_{g011}$) for this system at 35MW load is shown in Figure 8. Table III summarizes the expected values of various performance quality and reliability indices for this system.

The discrete probabilities of Figure 7 for $Q_{g110}$ at 115MW load reveal that the highest probability is associated with generation bottling of 15MW and not with zero as would be expected for well-designed systems. This indicates a general weakness (shortage) in the installed transmission facilities as compared with the existing generation capacities and required load. The discrete probabilities of Figure 8 for $Q_{g011}$ show two distinct high probabilities of occurrence at both 0 and 50MW surplus levels. This is mainly due to the fact that most of the main facilities (generators and transmission lines) are available (outage does not occur) which means that the network could deliver all the available generation (not needed) through transmission lines to the load side. On the other hand, at $Q_{g011}=0$, most of the main facilities (generators and transmission lines) are not available (outage), especially G1, T1 and T2, meaning that the network either doesn’t have the capacity of generation over the required load or it is incapable to deliver the available generation (not needed) through transmission lines to the load side. In this regard, the maximum value of the probability density of the $Q_{g111}$ for the system occurs at 75MW load level as shown in Figure 8.
### TABLE II: PERFORMANCE QUALITY INDICES FOR THE TEST SYSTEM AT 35MW LOAD

<table>
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<tr>
<th>#</th>
<th>G1</th>
<th>G2</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
<th>∑Pφ</th>
<th>∑Pκ</th>
<th>LNS</th>
<th>Q_{111}</th>
<th>Q_{116}</th>
<th>Probability</th>
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<th>eQ_{116}</th>
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<td>52</td>
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</table>

### IV. CONCLUSIONS

A general and coherent formulation is presented which can be used to account for the randomness associated with the load level and the availability of generation and transmission capacities. The general probability formulation can be used to calculate various reliability indices and quality measures. The paper describes the new approach for computing the probabilistic evaluation (expected value) of the reliability indices and performance quality measures and presents
The expected values of reliability indices, such as the Expected Load Not-Served (eLNS) and the expected values of performance quality indices: Utilized Generation Capacity ($eQ_{p10}$), Bottled Generation Capacity ($eQ_{p11}$), Shortfall Generation Capacity ($eQ_{p13}$), Deficit Generation Capacity ($eQ_{p01}$), Surplus Generation Capacity ($eQ_{p00}$), Redundant Generation Capacity ($eQ_{p09}$), Saved Generation Capacity ($eQ_{p08}$), and Surplus Generation Capacity ($eQ_{p07}$) are calculated in this paper with the established flow pattern, based on the solution of the basic linear program.

<table>
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<tr>
<th>Index</th>
<th>eLNS</th>
<th>$eQ_{p11}$</th>
<th>$eQ_{p13}$</th>
<th>$eQ_{p01}$</th>
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<tr>
<td>Value (MW)</td>
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<td>43.56</td>
<td>12.90</td>
<td>6.85</td>
<td>0.99</td>
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The general framework and formulation introduced in this paper can be applied to practical power systems. Also, it can be applied to a system under normal operation or subject to contingencies with certain or random occurrences. In this paper, the optimization technique based on three metaphors (dimensions) representing the relationship between available generation capacities and required demand levels was utilized. The novel formulation presented in this paper can be applied to practical power systems. Also, it can be applied to a system under normal operation or subject to contingencies with certain or random occurrences. The expected values of reliability indices and the expected values of performance quality measures.

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REFERENCES


