

# Comparative Study of Fatigue Damage Models Using Different Number of Classes Combined with the Rainflow Method

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**Abstract**— Fatigue damage increases with applied load cycles in a cumulative manner. Fatigue damage models play a key role in life prediction of components and structures subjected to random loading. The aim of this paper is the examination of the performance of the “Damaged Stress Model”, proposed and validated, against other fatigue models under random loading before and after reconstruction of the load histories. To achieve this objective, some linear and nonlinear models proposed for fatigue life estimation and a batch of specimens made of 6082T6 aluminum alloy is subjected to random loading. The damage was cumulated by Miner’s rule, Damaged Stress Model (DSM), Henry model and Unified Theory (UT) and random cycles were counted with a rain-flow algorithm. Experimental data on high-cycle fatigue by complex loading histories with different mean and amplitude stress values are analyzed for life calculation and model predictions are compared.

**Keywords**- damage models; damaged stress model; performance; random loading

## I. INTRODUCTION

Methods for fatigue-life assessment can be divided into two groups. One of them includes algorithms based on numerical methods of cycle counting [1- 6] and the other group uses spectral analysis of stochastic processes [3, 5]. In the first group, the material loading is represented by stress or strain histories, and in the second group it is represented by their frequency characteristics, i.e., power spectral density functions [6].

Engineering structures are often subjected to complex loads that undergo cyclic changes in time. They can be regular, when amplitudes and cycle shapes remain constant or irregular with stepwise changes in amplitudes or cycle shapes (so-called block loading), as well as stochastic, when amplitudes or cycle shapes vary in a random manner. Cumulative fatigue damage is an old, but not yet resolved problem. In general, cumulative fatigue damage theories can be classified into two categories: (1) linear damage cumulative theories and (2) nonlinear damage cumulative theories.

More than eighty years ago, in order to predict residual life, Palmgren suggested the concept which was later popularised by Miner [7] and is known as linear Miner rule expressed in a mathematical form as:

$$D = \sum_{i=1}^n \frac{n_i}{N_{Ri}} = \sum_{i=1}^n \beta_i \quad (1)$$

where  $n_i$  is the number of cycles at a given stress amplitude,  $N_{Ri}$  is the number of cycles failure at the same stress amplitude and  $D$  is the damage variable (it is equal to 1 in correspondence of exhausted life).

Many researchers have tried to modify Miner's rule, but, due to its intrinsic deficiencies, no matter which version is used, life prediction based on this rule is often unsatisfactory [8]. Richart and Newmark [31] introduced the concept of the damage curve to overcome the deficiencies associated with the linear damage rule. Upon this concept and the results of load sequence experiments, Marco and Starky [9] proposed the first nonlinear load dependent damage theory, represented by a power law:

$$D = \sum_{i=1}^n (\beta_i)^{x_i} \quad (2)$$

where  $x_i$  is a variable quantity related to the loading level.

This rule allows taking correctly into account the effects of different loading sequences. Experience has shown that only in some case and for some materials this law, and the other theories derived from it, have shown good agreement with experimental results. Moreover, the involved coefficients  $x_i$  have to be calculated for each different load and load condition limiting their use in engineering applications [10].

Damage theories based on endurance limit reduction have also been developed. Kommers and Bennett [32,11] further investigated the effect of fatigue pre-stressing on endurance properties using a two level step loading method. Their experimental results suggested that the reduction in endurance strength could be used as a damage measure.

Henry [12] is the first who correlated the damage parameter to life fraction. This model does not take into account the load interaction effects. With some assumptions [38], Henry showed that damage  $D$  stored by a test bar may be defined as the relative variation of the fatigue limit:

$$D = \frac{\sigma_D - \sigma'_D}{\sigma_D} \tag{3}$$

Expression of damage proposed by Henry is written as:

$$D = \frac{\frac{n}{N}}{1 + \frac{\sigma_D \left(1 - \frac{n}{N}\right)}{\sigma - \sigma_D}} \tag{4}$$

Or, if we set  $\gamma = \frac{\sigma - \sigma_D}{\sigma_D}$  = stress ration and  $\beta = \frac{n}{N}$ ,

$$D = \frac{\beta}{1 + \frac{1}{\gamma}(1 - \beta)} \tag{5}$$

where  $n$  is the applied number of cycles and  $N$  the number of cycles to crack initiation in the Wöhler curve.

It is noted that for  $\sigma'_D = 0$ , we have  $D = 1$  and that, when  $\sigma \rightarrow \sigma_D, \gamma \rightarrow 0$  and  $D \rightarrow 0$ .

Early theories accounting for load interaction effects were developed by Corten-Dolon [13] and Freudenthal [14, 15]. Both theories are based on the modification of the S-N diagram.

Bui-Quoc, [16, 17] developed a theory named Unified Theory that used the concept of Henry [12] and Gatts [18] combined with some parameters of Shanley [19] and Valluri [20] theories. Unified theory is based on the representation of the damage by reducing the endurance limit of the material; it is formulated as a relation of the type:

$$\left(\frac{\sigma_D}{\sigma_{D0}}\right) = \left(\frac{R_m}{R_{m0}}\right)^m \tag{6}$$

The endurance limit decrease, according to the number of cycles, following the form:

$$\frac{d\left(\frac{\sigma_D}{\sigma_{D0}}\right)}{dn} = -\frac{1}{a} \left(\frac{\sigma_D}{\sigma_{D0}}\right)^b \left(\frac{\sigma}{\sigma_{D0}} - \frac{\sigma_D}{\sigma_{D0}}\right)^2 \tag{7}$$

where  $a$  and  $b$  are constants of the material.

The damage is suggested by Bui-Quoc [21] as

$$D = \frac{\beta^s}{\beta^s + (1 - \beta^s) \frac{\lambda_2 - (\lambda_2 / \lambda_f)^s}{\lambda_2 - 1}} \tag{8}$$

$$\lambda_f = \frac{\sigma_u}{\sigma_D}; \quad \lambda_2 = \frac{\sigma}{\sigma_D}$$

where  $s$  is a parameter which is determined by the loading condition.

A new nonlinear approach, based on damage mechanics, has been proposed with the original concepts developed by Kachanov [39] and Rabotnov [40]. Lemaitre [22, 23] applied these principles to formulate a nonlinear damage evolution equation in the form:

$$\delta D = \left[1 - (1 - D)^{\beta_c + 1}\right]^\alpha \left[\frac{\sigma_a}{M_0(1 - D)}\right]^{\beta_c} \delta n \tag{9}$$

Where  $\beta_c, M_0$  and  $\alpha$  are coefficients of the Chaboche model.  $\beta_c$  and  $M_0$  depend on material and  $\sigma_a = \sigma_{max} - \sigma_m$  ( $\sigma_{max}$  and  $\sigma_m$  are respectively the maximum and the mean stress of cycle); the exponent  $\alpha$  depends on the loading ( $\sigma_{max}; \sigma_m$ ) which results in non-separability between damage and loading.

Recently, a model was suggested and validated by Mesmacque [24] and Aid [25, 26, 27, 33, 41] that does not require too many properties of the material and it takes into account the loading history. This model is connected cycle by cycle with Wohler's curve. The damage is calculated with the relation

$$D_i = \frac{\sigma_{(i)d} - \sigma_i}{\sigma_u - \sigma_i} \tag{10}$$

where  $\sigma_{(i)d}$  is the damaged stress;  $\sigma_i$  is the applied stress, and  $\sigma_u$  is the ultimate stress.

The indicator of damage at level  $i$  is calculated from (9), then it is transposed to the level  $i+1$  by the relation:

$$D_i = \frac{\sigma_{equiv} - \sigma_{i+1}}{\sigma_u - \sigma_{i+1}} \tag{11}$$

Where  $\sigma_{equiv}$  is the equivalent stress of damage at the level  $i+1$  and  $\sigma_{i+1}$  is stress at the level  $i+1$ . For more details refer to [25-27].

The experimental observations highlighted that when a positive average stress,  $\sigma_m$ , is superposed on the cyclic loading of amplitude the life of the specimens decreases. The inverse effect is observed in the presence of average stress of compression [42].

In order to take into account this phenomenon, complementary tests can be carried out in order to establish Haigh's diagram, for lifetime fixed at  $N_R$ , the alternate equivalent stress  $\sigma_{aeq}$  according to the characteristics ( $\sigma_a, \sigma_m$ ) of the cycle. This returns in fact to modify the curve of Wöhler for each cycle with  $\sigma_m \neq 0$ .

The real loadings are often more complex. Estimation of fatigue life of a material subjected to random loading needs suitable calculation algorithms. In such algorithms, cycle counting [28] according to a given method (for example, the

Rain Flow method) and damage accumulation according to the assumed hypothesis are the main operations.

In this investigation, Miner’s rule, Henry Model, Unified Theory and Damaged Stress Model are retained to damage accumulation and lifetime prediction of components subjected under random loading.

II. MATERIAL AND METHOD

A. Material, shape of the specimens and loading conditions

The experiments were conducted on the aluminum alloy 6082-T6, its chemical composition and principal mechanical properties are respectively presented in Table I and Table II. The tests were carried out on planar specimens, which were cut from a plate with a 8 mm thick (see Figure 1).

The fatigue tests were carried out using a 100 kN capacity Instron Servo-hydraulic machine 8500 for tests of planar specimens made of aluminum alloy 6082T6 under uniaxial tension–compression. While testing, the level of sinusoidal loading amplitude was controlled at zero mean value and 30 Hz frequency. The experiments were carried out under uniaxial constant amplitudes for determining the Wohler’s curves modeled by (12) according to ASTM standards [29]:

$$\text{Log}(\sigma) = A - m \cdot \text{log}(N_R) = 6.8 - 0.1024 \cdot \text{log}(N_R) \quad (12)$$

where  $N_R$  is the number of cycles to fracture,  $\sigma$  is the stress amplitude and  $m$  is the exponent of the Wohler’s curve. The fatigue limit of our material at  $N_R = 2.10^6$  is found to be  $\sigma_f = 220$  MPa.

TABLE I. NOMINAL CHEMICAL COMPOSITION OF THE 6082T6 ALUMINIUM ALLOY

Element	Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Al
Mean %	0.7	0.24	0.06	0.9	0.6	0.02	0.06	0.02	Rest

TABLE II. MECHANICAL PROPERTIES OF THE 6082T6 ALUMINIUM ALLOY

Sy (MPa)	UTS (MPa)	E (GPa)	A (%)
347	370	73	12

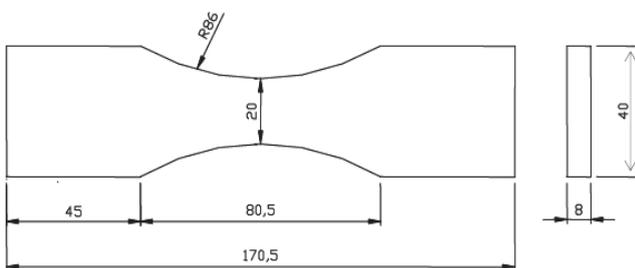


Fig. 1. Fatigue specimens-dimensions in millimeters

B. Random loading

The specimens were tested under random spectrum (original spectrum generated between -1 and +1), (see Figure

2), with different mean value of stress and various standard deviations (STD) values. Densities and the stress distributions for spectra, whose characteristics defined, are represented in Figures 3 and 4. Cycles were counted using the rain-flow method for numbers of classes equal to 11, 20, 25, 32, 40, 50 and 64.

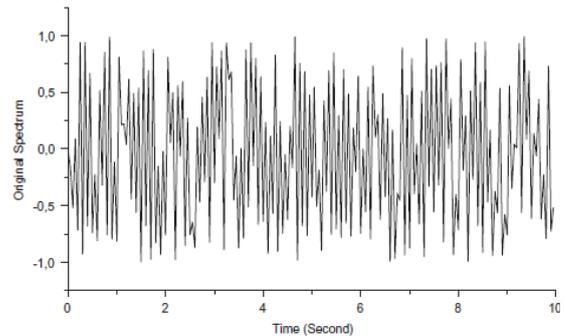


Fig. 2. Original load history.

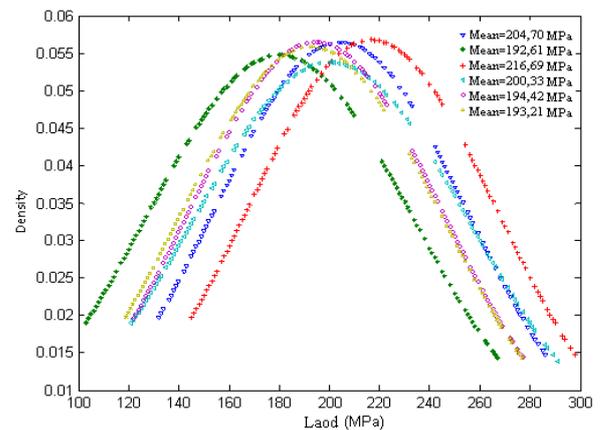


Fig. 3. Load density with constant standard deviation (STD) and different mean value.

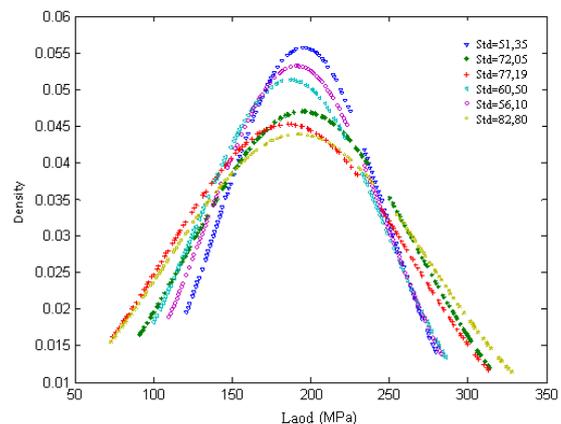


Fig. 4. 4. Load density with constant mean value and standard deviation (STD).

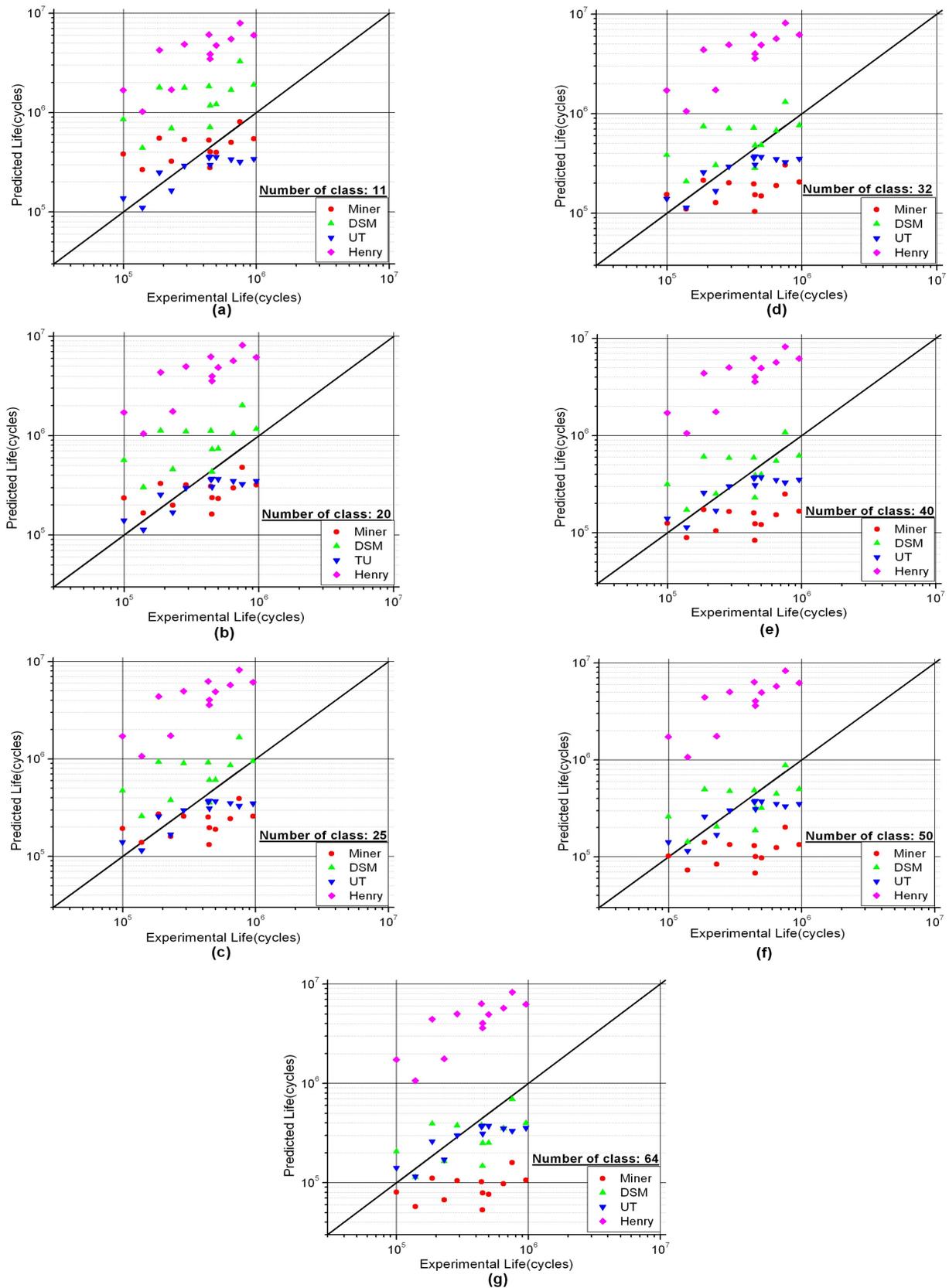


Fig. 5. Comparison between theoretical results and experimental results for different number of classes ((a) 11, (b) 20, (c) 25, (d) 32, (e) 40, (f) 50, (g) 64).

### III. RESULTS AND DISCUSSION

Reconstruction techniques that create the exact original history from some compact characterization have not been achieved due to complex algorithms and large storage requirements [30]. Reconstructions from statistical methods produce good results for random histories and, therefore, reconstructions based on cycle counting must be employed [43]. Hence, reproduction of histories that yield damage equivalent to the original history, without necessarily being identical to the original history, is desirable [34]. Reconstructions from simple cycle counting methods may produce histories with damage characteristics different from the original history. This problem arises from shortcomings in the cycle counting techniques [2]. Reconstructions from the rain-flow technique, however, always produce the same load cycles. Only the ordering of the cycles is different.

Life prediction and cycle counting are independent of each other. Any pairing of the two procedures will yield a life prediction because any life prediction model requires cycle information for input. Hence, fatigue life may be predicted once the cycle counting is performed. Since the rain-flow cycle counting procedure is used for load spectrum regeneration, it is also used as an input to the life prediction model, as a matter of convenience. This is desirable; however, the rain-flow method typically predicts life better than the simpler cycle counting methods [35].

The experimental results and different model's (Miner, DSM, Henry and Unified Theory) life prediction under random loading, in the above conditions and for cycles count by a rain-flow algorithm with different number of classes are compared. Comparison for a number of classes equal to 11, 20, 25, 32, 40, 50 and 64, are shown in Figure 5.

The prediction results show the following:

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### VI. CONCLUSION

This study is conducted to show that the proposed model (DSM) is nonlinear and takes into account the loading history. The proposed model correctly follows the experimental results and takes into account the effect of loading history. It is based only on the S-N (Wohler) curve which has to be known just for a part of the loading conditions; the unknown part can be estimated by a wide range of models. The results show that the proposed model is affected by the change of the number of classes and the predictions are improved if the number of classes tends to standards. The result obtained from the model are in good agreement with experimental results.

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