Reliability Assessment of Steel Plane Frame's Buckling Strength Considering Semi-rigid Connections

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Abstract—**The buckling strength of a structure depends hugely on material properties, geometry dimensions, and boundary conditions that are potentially random. This paper presents an assessment of the safety of buckling strength of steel plane frame semi-rigid beam-column connections. The Newton-Raphson method is used to solve the nonlinear equation for the buckling limit of the column. The reliability of the structure is evaluated by the Monte Carlo simulation method. The effects of input parameters are also investigated.**

Keywords-plane frames steel; semi-rigid connection; reliability; Monte Carlo simulation

I. INTRODUCTION

The analysis of steel frame with semi-rigid connections is always an interesting research topic. Straight-line common beam method was used to classify semi-rigid connection in [1]. Authors in [2] considered the stiffness of the connection in the research on the moment distribution method. Author in [3] investigated the influence of semi-rigid connections on the frame subjecting to shake. Research and application of the stiffness matrix method to analyze the plane frame with semi-rigid connections, in which the stiffness matrix and nodal force vector of each element depend on the linear stiffness of the connection, was conducted in [4]. A novel method to analyze the spatial frame considering the material nonlinearity and large deflection effect was presented in [5]. Linear and nonlinear semi-rigid connection models have been presented in [6]. There are many studies about buckling analysis of steel structures considering semi-rigid connection, like the in-plane buckling behavior of pitched-roof steel frames with semi-rigid connections and elastic buckling [7], second-order behavior of pitched-roof steel frames [8], and the buckling analysis of non-prismatic columns using slope-deflection method [9]. The buckling and post-buckling behavior of gabled frames was studied in [10]. Stability analysis of semi-rigid composite frames has been studied in [11]. Stiffness analysis on semi-rigid joints in gabled frames was studied in [12], and the elastic buckling of end-loaded, tapered, cantilevered beams with initial curvature in [13]. In this research, buckling analysis of steel gabled frames with tapered members and flexible connections was used.

Concerning the reliability assessment of the structure, authors in [14] assessed the reliability of steel frame under dynamic loads generated from the El Centro earthquake in 1940 using the β probability index method. The same method was used in [15] to study the steel frame according to the plastic limit state. The reliability of the steel structure under corrosion was studied in [16]. The Monte Carlo simulation method to investigate steel frame with flexible beam-column joint was utilized in [6]. Rotational stiffness and moment-resisting of the connection were referred in [17]. This model of flexible joint is not correct [18]. Instead a novel model based on empirical tests and Eurocode-3 was proposed.

This paper performs reliability assessment of the buckling strength for steel plane frame considering semi-rigid connections. An algorithm using Monte Carlo simulation is proposed to be used in analysis and assessment. In addition, the effects of input parameters, which are safety factor and coefficient of variation, on the reliability of structures have been examined.

II. THEORETICAL FRAMEWORK

A. Stiffness of the Beam-column Joint

The rotational stiffness of flexible beam-column joint of an I-section steel portal frame is determined by the experimental formula proposed in [18]:

$$\bar{\varphi} = \frac{\varphi_{ini}}{\eta} \qquad (1)$$

where $\bar{\varphi}$ is the elastic rotational stiffness of the joint (kNm/rad), η is the hardness adjustment coefficient depending on the connection type and presented in Eurocode-3, φ_{ini} is the initial stiffness (kNm/rad) determined as [18]:

$$\varphi_{ini} = K_1\, h_c^{0.44}\, h_b^{1.2}\, t_p^{0.35}\, d^{0.005} - K_2 \qquad (2)$$

where h_c (mm) is the height of the column section (HEB), h_b (mm) is the height of the beam section (IPE), t_p (mm) is the thickness of the end plate, and d (mm) is the bolt diameter,

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and K_1=1.5 and K_2=19211 are empirical coefficients from the experimental results in [18]. Because of their empirical origin, these coefficients possess potential randomness.

B. Monte Carlo Simulation Method

Monte Carlo simulation method is based on the use of pseudo-random numbers and the law of large number to assess the reliability of any system. If the safe domain is defined by the condition $f(\boldsymbol{X})>0$, where $\boldsymbol{X}$ is a random vector containing all the input random variables, the unsafe probability of the system is determined by:

$$P_f=\int I_{f(\mathrm{X})<0} f_{\mathrm{X}}(x)\,dx=E\left[I_{f(\mathrm{X})<0}\right] \quad (3)$$

where $I_{f(\mathrm{X})<0}$ is the indicator function defined by:

$$I_{f(\mathrm{X})<0}=\begin{cases}1 & \text{if } f(\mathrm{X})<0\\ 0 & \text{if } f(\mathrm{X})\geq 0\end{cases} \quad (4)$$

According to the theory of statistics, if we have N realizations of the random vector $\boldsymbol{X}$, by propagating the randomness, we obtain a sample of N realizations of the indicator function. The expected value of the indicator function can be approximatively determined by taking the mean of the samples:

$$\hat{P}_f=E\left[I_{f(\mathrm{X})<0}\right]=\frac{1}{N}\sum_{i=1}^{N} I^{i}_{f(\mathrm{X})<0} \quad (5)$$

A 95% confidence interval of the estimation is defined by [20]:

$$\hat{P}_f\left(1-200\sqrt{\frac{1-\hat{P}_f}{N\hat{P}_f}}\right)\leq P_f\leq \hat{P}_f\left(1+200\sqrt{\frac{1-\hat{P}_f}{N\hat{P}_f}}\right) \quad (6)$$

C. Displacement Method

This method is commonly used when analyzing stability structures by analytical method. This method is determined in [19] and the stability equations are shown by:

$$\det\begin{bmatrix} r_{11} & \cdots & r_{1j}\\ \vdots & \vdots & \vdots\\ r_{i1} & \cdots & r_{ij}\end{bmatrix}=0 \quad (7)$$

where r_{ij} is the reaction from a generalized relocation of ties.

III. Methodology

A. Reliability of Buckling Strength of the Steel Plane Frame Considering Semi-rigid Connections

1) Safe Condition

Although the buckling strength of the steel plane frame considering semi-rigid connections beam-column is determined by (13), in reality we often introduce a safety factor n and the safe condition of the steel plane frame considering semi-rigid connections beam-column will be written as:

$$P\leq\frac{P_{cr}}{n} \quad (8)$$

where P is the external load which can fluctuate randomly.

2) Deterministic Model and Uncertainty Model

The deterministic model deals with the above buckling analysis problem, in which the input parameters are those of geometry $\left(L_1,L_2,H_1,H_2,h_c,b_{fc},t_{fc},t_{wc},h_b,b_{fb},t_{fb},t_{wb},t_p,d\right)$, material ($E$) load ($P$) and flexible joint ($K_1$, K_2). This model can be written as:

$$P_{cr}=\Im(\boldsymbol{X}) \quad (9)$$

with $\boldsymbol{X}=\left[L_1,L_2,H_1,H_2,h_c,b_{fc},t_{fc},t_{wc},h_b,b_{fb},t_{fb},t_{wb},t_p,d,E,P,K_1,K_2\right]$. In this paper, we consider the steel plane frame considering semi-rigid connections with the deterministic input parameters shown in Table I.

The uncertainty model is constructed based on the deterministic model by taking into account the randomness of some input paramters. In this paper, we distinct two vectors of input parameters: the first one of the parameters assumed to be deterministic $\boldsymbol{X}_1=\left[L_1,L_2,H_1,H_2,h_c,b_{fc},t_{fc},t_{wc},h_b,b_{fb},t_{fb},t_{wb},t_p,d\right]$ and the second one of the parameters assumed to be ramdom $\boldsymbol{X}_2(\omega)=\left[P(\omega),K_1(\omega),K_2(\omega),E(\omega)\right]$ with ω representing the randomness of the parameters. This model can be written as:

$$P_{cr}(\omega)=\Im\left(\boldsymbol{X}_1,\boldsymbol{X}_2(\omega)\right) \quad (10)$$

TABLE I. DETERMINISTIC INPUT PARAMETERS

Variable	Value	Variable	Value
H_1 (cm)	400	L_1 (cm)	400
H_2 (cm)	800	L_2 (cm)	600
h_c (mm)	400	h_b (mm)	400
b_{fc} (mm)	200	b_{fb} (mm)	200
t_{fc} (mm)	15	t_{fb} (mm)	15
t_{wc} (mm)	10	t_{wb} (mm)	10
t_p (mm)	20	d (mm)	22

TABLE II. RANDOM INPUT VARIABLES AND THEIR REPRESENTATIVE PARAMETERS

Random variable		$P(kN)$	
Law of probability		Norm	
Representative parameters		μ_P	CV_P
		1.5088×10^4	0.1
Random variable		E(kN/cm^2)	
Law of probability		Norm	
Representative parameters		μ_E	CV_E
		2×10^4	0.1
K_1		K_2	
Uniform		Uniform	
Reference	Interval	Reference	Interval
1.5	[0.95÷1.05]	19211	[0.95÷1.05]

B. Column Reliability Assessment by Monte Carlo Simulation

By introducing the uncertainty model in the Monte Carlo simulation method, we obtain the scheme of the reliability assessment of the steel plane frame considering semi-rigid connections shown in Figure 1.

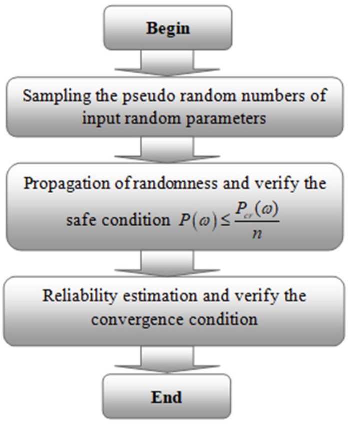

Fig. 1. Scheme of the reliability assessment of the steel frame considering semi-rigid connection using Monte Carlo simulation method

IV. NUMERICAL RESULTS

A. Validation of the Steel Frame Buckling Strength Considering Semi-rigid Connections

In order to validate the Matlab code, the steel frame considering semi-rigid connection shown in Figure 2 was considered with the input parameters shown in Table III. The validation of the program was performed in articulation joints (stiffness of the joint is zero) and rigid joints (stiffness of the joint is infinitely high). The result of the computer program and its comparison with the result of [19] is shown in Table IV.

TABLE III. INPUT PARAMETERS

Geometry and section details					
Beam	***L*** **(cm)**	**h_{wb} (cm)**	**b_{fb} (cm)**	**t_{fb} (cm)**	**t_{wb} (cm)**
	600	40.0	20.0	2.0	1.0
Column	***H*** **(cm)**	**h_{wc} (cm)**	**b_{fc} (cm)**	**t_{fc} (cm)**	**t_{wc} (cm)**
	600	40.0	20.0	2.0	1.0

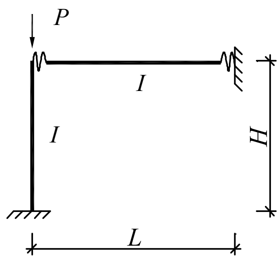

Fig. 2. Plane frame semi-rigid connections for validation

TABLE IV. VALIDATION BY COMPARISON WITH [19]

	Articulation joints			Rigid joints		
	Matlab code	**[2]**	**Error (%)**	**Matlab code**	**[2]**	**Error (%)**
P_{cr} (kN)	545.47	545.16	0.31	759.86	761.00	1.14

We can observe that the error is very small in the case of button joint (<0.31%) and slightly increases in the other case (<1.14%). This result can be explained by the fact that in our program the rigid joint is modeled by setting a very high value to the stiffness of the joints. This result confirms the reliability of our program.

B. Convergence of the Monte Carlo Simulation

Consider the steel plane frame semi-rigid beam-column connections shown in Figure 3 with the deterministic input parameters and the random input parameters presented in Table I and Table II respectively. The compression load *P* (kN) and Young' modulus are assumed to be normal variables with mean μ_P, μ_E and coefficients of variation CV_P, CV_E, whereas the empirical coefficients K_1, K_2 are assumed to be uniform variables in the interval [0.95, 1.05] of the reference value [18]. Figure 4 shows the convergence of the safe probability of the frame in the Monte Carlo simulation to the value of 0.9312 or 93.12% after about 1051 samples in 30 minutes. The used convergence criterion of 2.5% justifies the confidence of the estimated reliability. This result also shows that although the safety factor was taken as 1.1 in the analysis, because of the randomness of some input parameters the reliability of the structure is only 93.12%. Thus the assessment of the reliability of the structure is necessary.

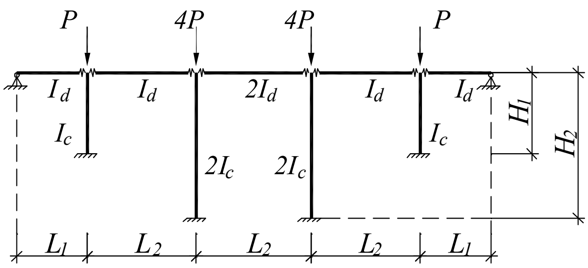

Fig. 3. Semi-rigid connection plane frame analysis

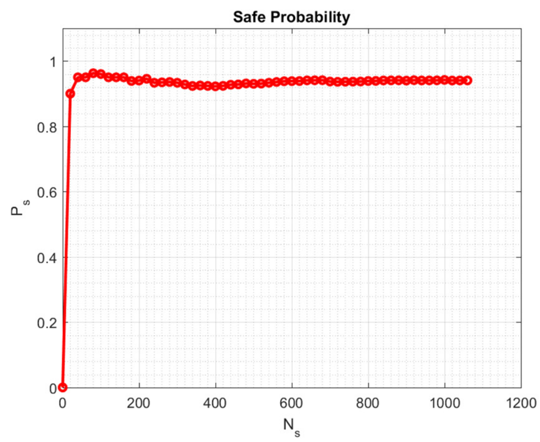

Fig. 4. Convergence of the safe probability in the Monte Carlo simulation

C. Coefficient of Variation and Safety Factor Effects

We know that the variation of input random variables and the safety factor influence directly but inversely the safe probability of the structure. Thus in order to clear the effect of these parameters, one reconsiders the above column with different coefficients of variation of the compression load CV=0.05; 0.1; 0.15; 0.2; 0.25 and different safety factors n=1.1; 1.15; 1.2; 1.25; 1.3. The randomness of the empirical coefficients K_1, K_2 is assumed to be unchanged in the interval [0.95, 1.05] of the reference value. The results of the safe probability are listed in Table V and presented in Figure 5.

TABLE V. EFFECTS OF THE COEFFICIENT OF VARIATION AND OF THE SAFETY FACTOR ON THE SAFE PROBABILITY OF THE FRAMES

n	***CV***				
	0.05	**0.10**	**0.15**	**0.20**	**0.25**
1.10	0.0276	0.1514	0.2566	0.2833	0.3500
1.15	0.0000	0.0676	0.1302	0.2222	0.2553
1.20	0.0000	0.0225	0.1166	0.1313	0.2166
1.25	0.0000	0.0067	0.0572	0.0964	0.1722
1.30	0.0000	0.0000	0.0278	0.0609	0.1031

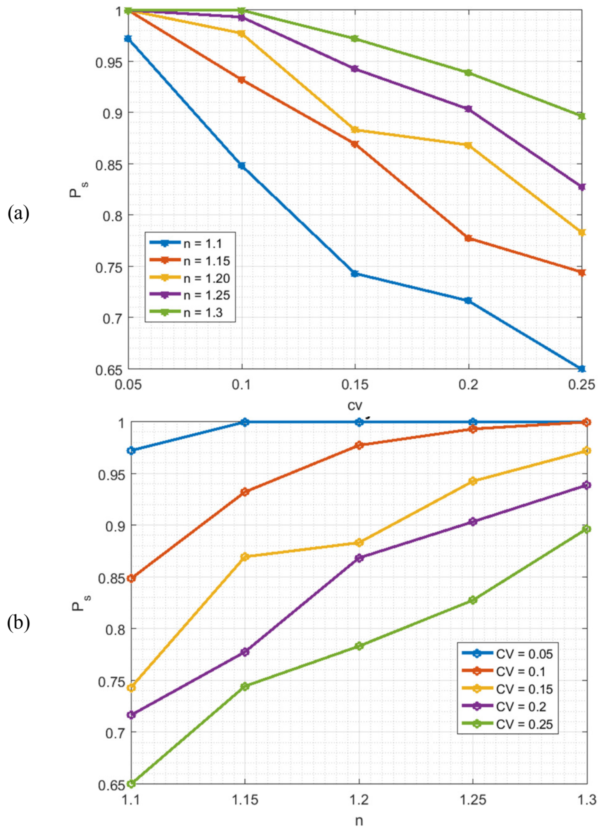

Fig. 5. (a) Effect of the coefficient of variation and (b) effect of the safety factor on the safe probability of the column

We can easily observe the inverse effect of the coefficient of variation and of the safety factor in Figure 5. The safe probability of the column decreases when the coefficient of variation increases, whereas it increases when the safety factors increase. This result seems to be obvious, but it has a very important significance. It shows that if there are many input random parameters or furthermore, with high randomness in the structural design or in the optimization problem, the use of local coefficients such as the overload coefficient seems to be not sufficient. The structure can be in a dangerous state. In this case, it is necessary to determine a global safety factor, as is done in this study, to assure the absolute safety of the structure. For example, in this test, if the coefficient of variation is 0.05, the global safety factor needs to be only 1.15 to obtain a safe probability of 100%. If the coefficient of variation is 0.1, the global safety factor needs to be 1.3.

V. CONCLUSIONS

This paper studies the reliability assessment of a steel plane frame with semi-rigid beam-column joint connections. The authors have successfully established the deterministic model using the Newton-Raphson method to solve the transcendental equation of the column's equilibrium. This model was then combined with the Monte Carlo simulation method to construct the column's reliability assessment program. Parametric tests were also performed to study the input parameter effects on the reliability of the structure. The obtained results show the significance of this research.

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