Effect of the Triaxiality in Plane Stress Conditions

Triaxiality Effect in a PVC material

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Abstract— **Polymer materials are gaining more and more importance in engineering applications. A new methodology of analysis is required in order to assess the capability of such material in withstanding complex loads. Therefore, the behavior of these materials currently arouses a great research interest. The use of PVC plastic pipes in pressure vessels and pipelines has increased rapidly in the last decade. In order to determine the plastic behavior of PVC, an experimental method is presented. Through the results obtained from experimental tests, in the first part of this paper, we investigate the use of a phenomenological model proposed by G'Sell and Jonas. The true stress-strain response under large plastic deformation was investigated in different stress triaxiality frameworks. Particular attention was given to volumetric strain evolution, separation resulting from elastic volumetric strain, plastic volumetric strain and pure shear. The effect of stress triaxiality on plastic instability and fracture strain was also examined. The deformation process should be considered as explained, and the anisotropic plastic response induced by the deformation could be introduced in constitutive equations of G'Sell.**

Keywords- Plane stress; Triaxiality; Plastic instability; PVC

I. INTRODUCTION

In the plastics industry, technical polymers are widely used in engineering components which may experience complex mechanical loadings. The understanding of their intrinsic mechanical behavior is of prime importance in the design of components made of such. Over recent years, considerable attention has been focused on the analysis of large plastic deformation of ductile materials and solid polymers. The deformation processes involved in the plastic deformation of the ductile materials have been widely investigated by several authors [1-17]. However, most studies conducted on solid polymers were based on the same criteria used for ductile materials [18–36].

The response of solid polymers is known to be sensitive to factors such as: temperature, strain rate, strain, plastic instability during the necking stage and the hardening, and the type of loading conditions, i.e. the stress triaxiality effect [21, 37-38]. However, the determination of these parameters and the characterization of materials are really difficult with traditional measuring equipment like the one used for ductile materials. Therefore, some experimental devices have been developed and proposed [25, 39-40].

The aim of this paper is to investigate the plastic behavior of Polyvinyl-Chloride (PVC) in different triaxiality frameworks, under large deformation and in plane stress condition. The PVC material used in this work is for pipe irrigation applications. We focused on the experimental determination of the constitutive behavior in well controlled conditions. The triaxiality effects induced by the flat sample shape are studied in detail. In recent years, the study and modification of the relationships of the stress triaxiality in plane stress and strain conditions, its evolution, the behavior law in ductile materials and fracture has been the subject of various studies [15, 41-47]. This work is based on the same principle, i.e. to validate the relationship of triaxiality proposed by Bai [43] for a polymer material in plane stress condition and in large deformation.

The PVC material presents a complex nature of behavior resulting to a difficulty in fine physical analysis. Therefore, the deformation and damage of ductile materials and polymers under different stresses has been investigated [11, 16, 47]. The use of a phenomenological law can provide a solution to the problem of predicting the behavior of such materials and also ductile fracture. Several models have been proposed in the literature in order to predict the behavior of ductile materials and polymers. These include several constitutive laws proposed [7, 8, 18, 48-56].

In this paper, the law of G'Sell and Jonas [18], where the stress depends on deformation, speed and temperature, is selected. This law has successfully characterized the behavior of a great number of polymers either amorphous or semi-crystalline, under different simple loading modes (traction or compression). In the present study, we introduce the use of this constitutive model to describe the mechanical behavior of PVC. In the last part of this paper, the large deformation behavior of PVC is examined. The relevant features which should be taken into account in the application of an accurate constitutive model (like the G'Sell model [22]), are discussed with a particular attention paid on the volumetric strain and damage.

II. MATERIAL AND EXPERIMENTAL METHODS

In this subsection, the investigated PVC material is described. Then, the specimens' geometry and the experimental procedure under different loading paths are presented.

A. Material

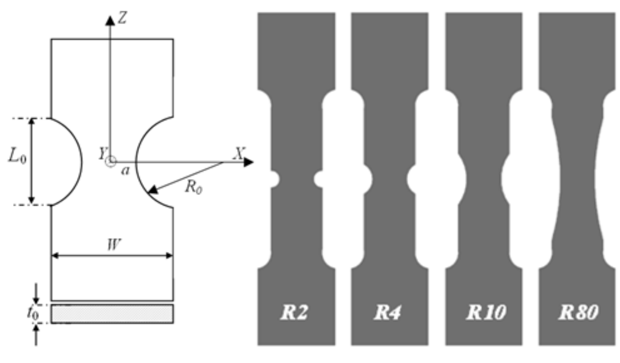
The polymer investigated in this work (PVC) is one of the most widely used and relevantly economical plastic materials, used for a large variety of industrial applications, especially in the field of packaging and of housing environment. In houses, it is used for pipings, dressing of windows and floor covering. PVC is a thermoplastic polymer with good strength, toughness and resistance to acids as well as corrosion.

In this study, the PVC material was provided by a piping company as pipes with outer and inner diameters of 200 mm and 212 mm, respectively. The physical properties of the material are: density: 1.38 g/cm³, melt mass flow: 1.2 g/10 min, molecular weight: 310000, glass transition temperature: 84 °C, melting point: 180 °C.

B. Specimens Geometry

In order to examine the sensitivity of true stress strain response on the behavior of the PVC material for various stress triaxiality, tensile tests were made on different geometries. For all tests, the samples were extracted from PVC pipes, and machined in the same extruded sheets in order to have a flat shape, with four different curvature radii on the ends of the width and in a symmetrical position (*R*= 2, 4, 10 and 80 mm). The specimens will be referred to as R_x, where x is the value of curvature radius. The initial dimension of flat specimens is defined by: the curvature radius R_0= 2; 4; 10; and 80 mm; the minimum width W_0=8 mm, when $W=2a$; the gauge length L_0; the thickness of the specimen t_0=4 mm and the maximum width $W=12mm$.

All flat specimens have the same minimum and maximum widths, as shown in Figure 1(a). In this type of specimen (flat in plane stress) with two symmetric cutouts, we can easily control the initial stress triaxiality. The geometry of the specimen is shown in Figure 1(b).



(a) (b)

Fig. 1. Flat grooved plane stress specimen: a) the cross section b) geometry of flat specimens.

One parameter that will be referred to frequently in this paper is the stress triaxiality β [59]. It is defined as the ratio of the mean stress σ_m and the von Mises equivalent stress σ_{eq} as:

$$\beta = \frac{\sigma_m}{\sigma_{eq}} \tag{1}$$

where σ_m is expressed as:

$$\sigma_m = \frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right) \tag{2}$$

It can be expressed, using the geometric parameters in the undeformed state, as the expression for the initial triaxiality, derived by Bai [43], is given by:

$$\beta = \frac{1+2A}{3\sqrt{A^2 + A + 1}} \tag{3}$$

where

$$A = \ln\left[1 + a/(2R_0)\right] \tag{4}$$

However, the stress triaxiality is not constant, but increases with plastic deformation, and the Bai formula is not valid in the median cross-section in large scale deformation [43]. Note that at the center of the median cross-section, the stress triaxiality ratio β reaches its maximum.

III. RESULTS AND ANALYSIS

A. Characterization of the plastic behavior

In this section, comparison will be made between the results of tensile tests carried out on flat specimens in plane stress conditions for different triaxiality. The objective of this analysis is to demonstrate the flat specimen applicability and also to investigate the effect of the triaxiality on the responses and fracture under large deformation. Tests were conducted on an INSTRON machine. An optical measurement system was used to control the strain rate and measure the local strain in the specimen section. The mechanical tests were achieved at a strain rate of 10^{-3} s^{-1} and at room temperature (23 °C). Figure 2 shows the evolution of the true axial stress and the volumetric evolution vs. true axial strain of the PVC material, and summarizes the stress triaxiality effect on the behavior and damage of this material. Globally, Figure 2(a) indicates that the response of this material is similar to other polymers investigated in the literature. In this response, the curve shows a proportional limit followed by a maximum at which necking takes place. It is common to term this maximum as the yield stress in polymer materials. Finally, before the rupture, and like most polymers, a hardening phenomenon is observed. Results show a strong influence of stress triaxiality on PVC behavior, more evident at large strains. The influence of stress triaxiality on the behavior is also evident in the evolution of volumetric strain, shown in Figure 2(b). The cavitation phenomenon increases with the increase in stress triaxiality. Indeed, it should be recalled that the multiaxial stress state in the median cross-section favors high positive hydrostatic stress that enhances cavitation damage, which itself induces a softening of the strain hardening rate.

In order to understand the macroscopic behavior of the PVC material, we have chosen to separate the volumetric strain according to the criteria described in [58]. We considered that the various changes in the volumetric strain evolutions are caused by linear elasticity and void volume evolutions coupled to shear deformation. We recall that the decomposition of deformation is given by the following relationship:

$$\varepsilon = \varepsilon_{Vel} + \varepsilon_{Vcav} + \varepsilon_{sh} \qquad (5)$$

where

$$\begin{cases} \varepsilon_{Vel} = \varepsilon_{el}(1-2\nu) \\ \varepsilon_{Vcav} = \varepsilon_V - \varepsilon_{Vel} \\ \varepsilon_{sh} = \varepsilon - \varepsilon_{Vcav} - \varepsilon_{Vel} \end{cases} \qquad (6)$$

and

$$\begin{cases} \varepsilon_{el} = \dfrac{\Sigma}{E} \\ \varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \ln\left(\dfrac{V}{V_0}\right) \end{cases} \qquad (7)$$

The true strain rate is maintained constant in the necking using an optical measurement system. At the beginning of the necking, a difference between the two deformations (transversal and longitudinal) will be detected by the optical system; therefore it will slow the speed of the machine in order to stabilize the local strain rate. The evolution of true stress-strain curves of the PVC material under uniaxial tension at a constant local true strain rate of 10^{-3} s^{-1} is presented in Figure 3 at different states of triaxiality according to the radius of curvature (*R2*, *R4*, *R10* and *R80*), respectively. A very ductile behavior in the PVC specimens whatever the triaxiality state, is shown. The true stress-strain behavior is characterized by four main stages: an initial elastic response, followed by a rollover to yield; a progressive plastic strain hardening (necking), and finally a significant plastic strain hardening at very large strains before final rupture. Also, the evolution of the three components of the volumetric strain according to the relationship of [57] (5) is presented.

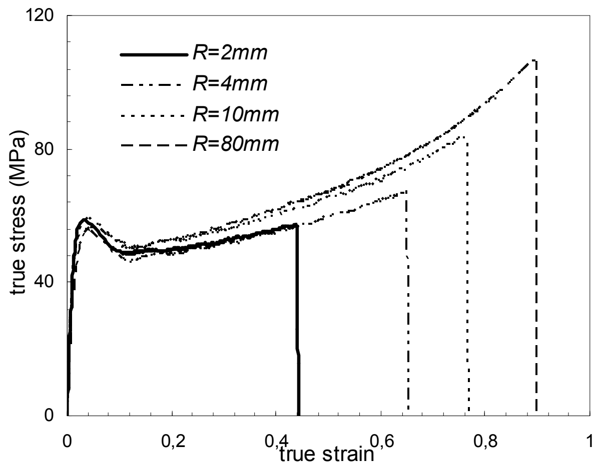
We noticed a typical character of the plastic behavior of polymer materials, which is characterized by a high ductility, and the triaxiality effect. The elastic volumetric strain ε_{Vel} is caused by the elasticity and the Poisson's ratio effect; then, the plastic volumetric strain ε_{Vcav} allows to characterize the onset of damage and its kinetics; the latter representing a significant proportion of the total deformation. Finally, the deformation due to pure plastic shear ε_{sh} allows us to appreciate the deformation involving no change in volume (Figure 3).

Another important feature is the influence of stress triaxiality on the plastic volumetric strain and the deformation due to pure plastic shear. The most direct indicator of this phenomenon is shown in a form of a pie chart by percentages.

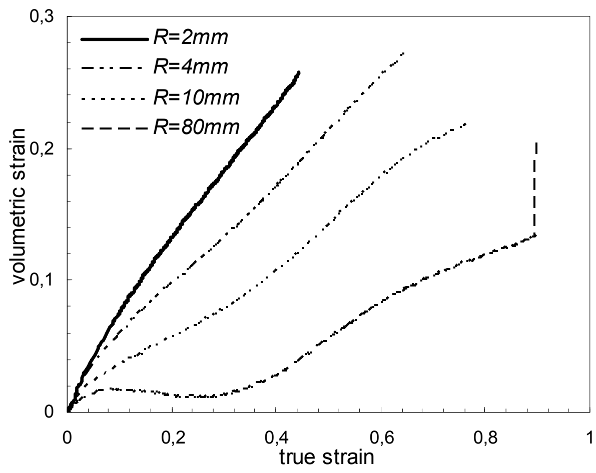
B. Plastic instability

In order to characterize the degree of inhomogeneity of deformation and identify the microstructural processes of the damage of the PVC material, the previously reported results are further investigated.

The true stress-strain curve shown in Figure 2(a) shows a viscoelastic range up to the yield point for all triaxialities considered in this study. During this stage, deformation is nearly homogeneous. At yield, several shear bands are hardly observed in the center of the sample and in the geometric defect as shown in Figure 4. After the yield point, we have observed a load drop with a decrease of true stress from 60 to 50 MPa. During this critical stage, the plastic yielding propagates across the entire width of the sample (in the center of the geometric defect). Finally, the true stress is stabilized; it appears that the center of the neck is less stable than before. Thus, a peculiar behavior for this polymer is documented: a necking phenomenon due to plastic instability (Figure 4).



(a)



(b)

Fig. 2. Effect of stress triaxiality on (a) stress-strain response and (b) volumetric strain evolution.

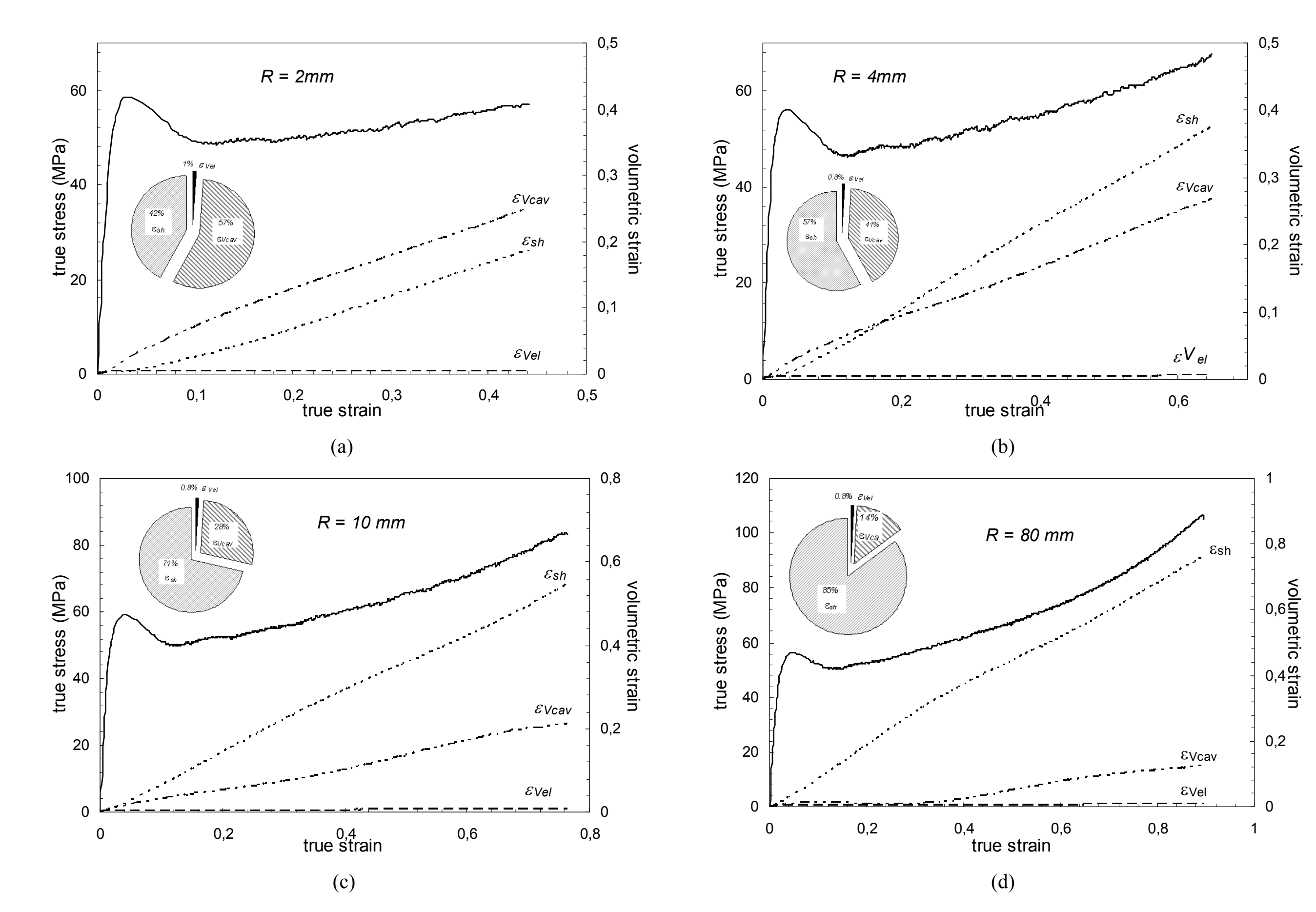


Fig. 3. True Stress and volumetric strain evolution vs true strain response.

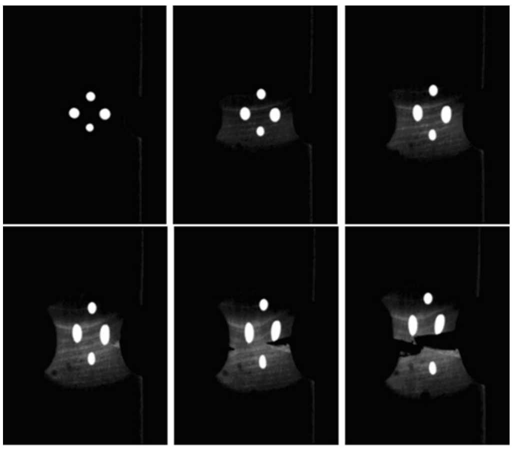


Fig. 4. Photographs of the stretching behavior under tensile testing.

The polymerists have sought more specific laws capable of accounting the viscoplastic behavior of most polymers. Several approaches have been proposed. As G'Sell and Jonas [18] have initially proposed, for PVC, a phenomenological model of behavior is given by (8) where σ is the equivalent stress, ε is the equivalent strain, m is the coefficient of sensitivity to strain rate and torque, h are two material constants:

$$\sigma\left(\varepsilon,\dot{\varepsilon}\right)=K\exp\left(h\varepsilon^{2}\right)\dot{\varepsilon}^{m} \qquad (8)$$

The factors $\exp\left(h\varepsilon^{2}\right)$ and $\dot{\varepsilon}^{m}$ respectively describe the hardening and viscous behavior.

G'Sell [22] has proposed to modify (8) by including others factors, describing the law of behavior of all polymers by a global relationship expressed as follows:

$$\sigma\left(\varepsilon,\dot{\varepsilon}\right)=K.f\left(\varepsilon\right).g\left(\dot{\varepsilon}\right) \qquad (9)$$

where K is a material constant and $f\left(\varepsilon\right)$ and $g\left(\dot{\varepsilon}\right)$ represent the influence of the deformation and the strain rate respectively.

The function $f\left(\varepsilon\right)$ can be written as:

$$f(\varepsilon)=V(\varepsilon).C(\varepsilon).H(\varepsilon) \qquad (10)$$

where
$$\begin{cases} V(\varepsilon)=1-\exp(-w.\varepsilon^{v}) \\ C(\varepsilon)=1+a.\exp(-b.\varepsilon) \\ H(\varepsilon)=\exp(h.\varepsilon^{n}) \end{cases} \qquad (11)$$

$V(\varepsilon)$ describes the viscoelastic behavior law; $C(\varepsilon)$ represent the draw-hook and $H(\varepsilon)$ describes the gradual hardening in large deformations, where n is equal to 2 in most cases.

After the verification of the rate sensitivity of the PVC material, we have observed that this material is not sensitive to strain rate; where this sensitivity is expressed by the function $g(\dot{\varepsilon})$, and can be written in the following form:

$$g(\dot{\varepsilon})=\dot{\varepsilon}^{m} \qquad (12)$$

In the context of this work, in order to understand and explain the mechanical behavior of the polymer studied, the G'Sell models [18, 22] are compared with the obtained experimental results of tensile testing. The found results after the identification of the parameters of the two formulas are presented in Figure 5. The curves show a good correlation between the experimental results and those from modeling. Indeed, the G'Sell law [22] gives the same evolution for the behavior of PVC and allows reproducing faithfully all phases of deformation for a constant strain rate. However, the G'Sell-Jonas law [18] does not reproduce the same experimental evolution and it does not consider the tensile hook that reflects the true response of the PVC. One thing very important on the G'Sell-Jonas model, is that it does not take into account the effect of stress triaxiality, this is why we documented some dispersion in the results between the extreme triaxialities of *R2* and *R80,* as shown in Figure 5.

In this paper we use the models without any modification. The introduction of the triaxiality effect in the hardening evolution will be the object of a future work. Another objective in this part of the paper is to verify the manifestations of plastic instability on PVC in tension with the effect of the triaxiality.

Under isothermal mechanical behavior and a constant strain rate we assume that the criterion of strain localization is:

$$dF = 0 \qquad (13)$$

with
$$\frac{d\sigma}{\sigma}=-\frac{dS}{S} \qquad (14)$$

and $V = SI$ representing the current volume. We obtain:

$$S = S_0\left(\frac{I_0}{I}\right)e^{\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}} = S_0 e^{\varepsilon_{22}+\varepsilon_{33}} \qquad (15)$$

$$dS = S_0 e^{\varepsilon_{22}+\varepsilon_{33}}\left(d\varepsilon_{22}+d\varepsilon_{33}\right) \qquad (16)$$

$$\frac{d\sigma}{\sigma}=-\left(d\varepsilon_{22}+d\varepsilon_{33}\right) \qquad (17)$$

where localized deformation conditions can also be expressed as follows:

$$\frac{d\sigma}{d\varepsilon_{11}}=-\sigma\left[\frac{d\varepsilon_{22}}{d\varepsilon_{11}}+\frac{d\varepsilon_{33}}{d\varepsilon_{11}}\right]=\overline{\sigma} \qquad (18)$$

The variations of the plastic instability versus the true strain are illustrated in Figure 5 for all triaxialities according to *R2, R4*, *R10* and *R80*, respectively. This evolution can be qualitatively related to the microstructural evolution involved in the plastic deformation of the PVC material. First, there is a stretching of chains, followed by rotation, plastic deformation, and finally the formation of a fibrillar microstructure and the significant strain hardening observed in the stress-strain response. It can be observed in Figure 2 that stress triaxiality has an important effect on strain hardening.

We recall that the critical value of the plastic instability is equal to 1, which means that a plastic instability less than 1 results to the development and propagation of necking, as observed experimentally (Figure. 4). However, it should be noted that the necking initial evolution stabilizes, which mean that plastic instability is greater than 1.

IV. Conclusion

The mechanical behavior under large plastic deformation of a PVC was studied experimentally under uniaxial tension by using a VideoTraction system. The plastic damage of this material was analyzed in different triaxiality frameworks. To this end, the material was tested in uniaxial tension on flat grooved specimens in order to set specific values of the stress triaxiality in the median cross-section in a plane stress conditions, and with an optical measurement system in order to control the strain rate and measure the strain in the local specimen section.

In order to quantitatively describe the damage of PVC, the volumetric strain was determined, with a decomposition caused by a linear elasticity and void volume evolutions coupled to shear deformation. It was found, for all tests, that the material undergoes significant volume changes during deformation favored by high values of the stress triaxiality ratio. The experimental results obtained in this work served fruitful basis of experimental data for introducing the use of a robust and constitutive model [18, 22]. To illustrate the applicability of this model to the mechanical behavior of PVC, a comparison was conducted between the two formulas. The complex mechanical behavior of polymeric materials can not be described by a simple constitutive law. This is why many models in the literature have shown deficiencies to describe the behavior of these materials. Indeed, the model of G'Sell [22] was able to reproduce in a simple and satisfactory way all the experimental results, and allows the description and reproduction of all phases of plastic flow such as

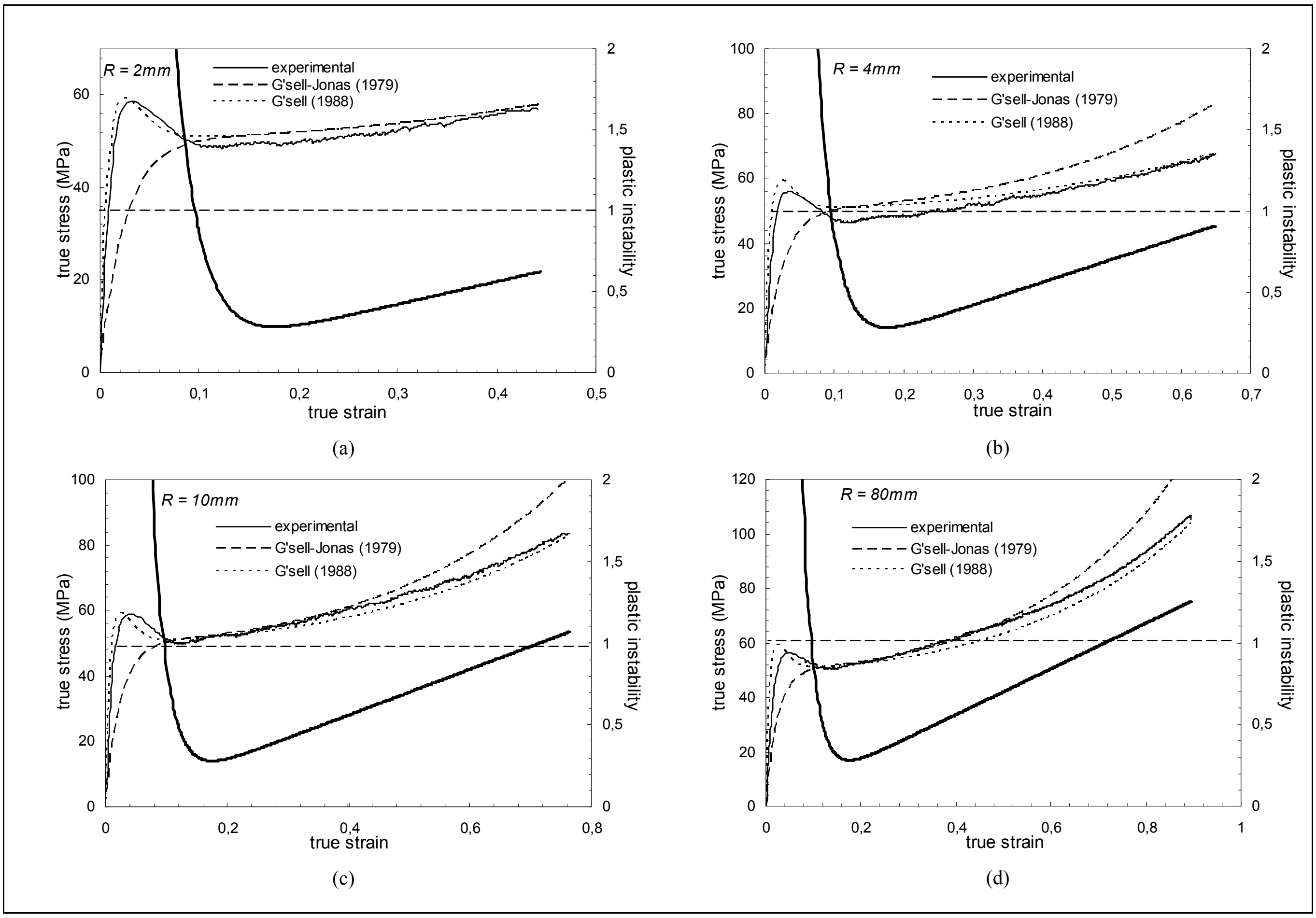


Fig. 5. True Stress and plastic instability vs true strain response.

draw-hook and gradual hardening in large deformations. In the continuum mechanics framework, the description of ductile materials is based on the concept of the yield criterion and its evolution with deformation. Since the studied material presents cavitation damage, this one is compressible. Therefore, the yield criterion needs to involve the hydrostatic stress but also the volumetric strain. Furthermore, the deformation process as explained in Figures 4 and 5 must be considered and at least the anisotropic plastic response induced by the deformation should be introduced in constitutive equations. In order to express quantitatively the damage kinetics in this material, we introduced the notion of 'plastic instability' which represents the instantaneous evolution of the volume strain vs true axial strain. Also it was found that plastic strain hardening increases gradually with the triaxiality. Finally, in order to highlight the evolution of the observed phenomena in the deformation process, e.g. volumetric deformation and plastic instability, the finite element modeling can prove very important. All these features will be investigated again in another paper on the basis of detailed characterization of plastically. However, approximate analytical theories, as those we presented in this work, remain very useful to account for basic phenomena.

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