Assessment of Wind Power Potential Based on Raleigh Distribution Model: An Experimental Investigation for Coastal Zone

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Abstract—The wind energy share in the global energy production is increasing rapidly. Currently, the Government of Pakistan (GoP) is moving towards renewable energy resources (RER), specifically wind and solar energy. In this paper, the wind energy potential of Tando Ghulam Ali, Sindh, Pakistan is explored. For this purpose, one-year wind speed data are considered at various heights through various probability distribution functions (PDFs). Statistical comparison of Rayleigh, gamma, generalized extreme value (GEV) and lognormal PDFs have been done with two methods, namely root mean square error and \( R^2 \) in order to select the best PDF. Results showed that the Rayleigh distribution function is the best at the above area for finding various factors like site selection and wind power cost per kWh.

Keywords—wind energy; wind probability distribution function; fitting tool

I. INTRODUCTION

Several countries are investing resources in the direction of RER. Energy sources like biomass, wind energy, hydro energy, and thermal energy have gained interest because of their friendly environment features [1, 21-23]. The wind energy is a drifting RER for production of energy, and can have better results over the reliability and stability of the power system. Wind power recorded a yearly development of 25% rate over the past few years [2, 20] and is based on the wind speed estimation, which is uncertain in considering wind energy and the performance of the wind energy conversion system. Wind speed at a location diverges unsystematically and its deviation in a specific area over a time period and can be characterized by PDFs. Several PDFs are taken to estimate the wind speed features including Rayleigh, Weibull, GEV, lognormal and gamma [3]. To find the most accurate of PDFs, prediction performance tests are suggested. In this paper, root mean square error (RMSE) and coefficient of determination \( R^2 \) tests are considered.

In [4], it was shown that the outcomes of the Weibull distribution dignified the PDF and are more suitable as compared to the Rayleigh distribution in the peak height region of Nepal [4]. Authors in [5] ranked 7 different methods considering the examined error of calculations regarding wind energy. Authors in [6] found the wind power density in a specific location of Pakistan at various heights and considered different distribution functions for wind speed calculation and from 5 numerical methods. Authors in [7] examined the comparison between the lognormal and Weibull techniques by fitting the curve of observed wind speeds and concluded that the Weibull distribution is more reliable than lognormal distribution to pronounce the performance of wind speed data. Authors in [8] explored the number of distribution techniques to find the exact wind speed data in Hong Kong. This study summarizes the results of wind speed data at Tando Ghulam Ali. This study tries to find the best distribution techniques from various PDFs like: Rayleigh, gamma, GEV and lognormal. From the results, it was proved that the Rayleigh distribution function was the best distribution technique for the studied site.

II. MATHEMATICAL PDFS

The estimation of wind energy at a particular site involves statistical methods of PDF, which require wind speed data at a meteorological station. Similarly, frequency distribution functions are used to estimate wind power density. Some types of PDFs are chi-squared distribution, Rayleigh distribution, generalized normal, three parameter log-normal, log normal-distribution, gamma distribution, kappa, wake by inverse Gaussian distribution, normal two variable distributions, hybrid distribution, as well as normal square root of wind speed distribution [9-11].
A. Rayleigh Distribution Model
Rayleigh distribution function [12]:
\[
f(v) = \frac{v^2}{\sigma^2} \exp \left( -\frac{v^2}{2\sigma^2} \right)
\]  
\[\text{(1)}\]
The shape parameter ($k$) is assumed to be 2 and the scale parameter ($\sigma$) is defined as:
\[
c = \frac{m}{r(1+\frac{2}{k})}
\]  
\[\text{(2)}\]
where ($m$) is the mean and can be obtained as:
\[
m = \sum_{i=1}^{N} v_i
\]  
\[\text{(3)}\]
B. Gamma Distribution Model
The gamma distribution function can be defined as [13]:
\[
f(v) = \frac{v^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} \exp \left( -\frac{v}{\beta} \right)
\]  
\[\text{(4)}\]
where,
\[
\alpha = \frac{m^2}{s^2}
\]  
\[\text{(5)}\]
\[
\beta = \frac{s^2}{m}
\]  
\[\text{(6)}\]
where ($s$) can be obtained as follows:
\[
s = \left[ \frac{1}{N-1} \sum_{i=1}^{N} (vi - m)^2 \right]^{\frac{1}{2}}
\]  
\[\text{(7)}\]
C. Lognormal Distribution Model
Lognormal distribution can be defined as [14]:
\[
f(v) = \frac{1}{\sigma \sqrt{2\pi}} \left[ \frac{1}{\sigma} \left( \log(v) - a \right)^2 \right]
\]  
\[\text{(8)}\]
Parameters from (8) are estimated as:
\[
\alpha = \ln \left( \frac{m}{\sqrt{1 + \frac{s^2}{m^2}}} \right)
\]  
\[\text{(9)}\]
\[
\beta = \sqrt{ \ln \left( 1 + \frac{s^2}{m^2} \right) }
\]  
\[\text{(10)}\]
D. Generalized Extreme Value Distribution (GEV) Model
It is a combination of Frechet, Gumbel and Weibull maximum extreme value distributions and can be defined as [15]:
\[
f(v) = \frac{1}{\alpha} \left[ 1 - \frac{\alpha}{\gamma} (v - \mu) \right]^{\frac{\gamma - 1}{\alpha - 1}} \left\{ \left[ 1 - \frac{\alpha}{\gamma} (v - \mu) \right]^{\frac{1}{\alpha - 1}} - \left( \frac{\gamma - 1}{\alpha - 1} \right) \right\} \text{if } k \neq 0
\]  
\[\text{(11)}\]
With the support of the maximum likelihood method, the parameters of GEV can be obtained as:
\[
LL = \ln \prod_{i=1}^{N} \left( e(v_i; \zeta, \delta, \theta) \right) = \sum_{i=1}^{N} \ln \left( e(v_i; \zeta, \delta, \theta) \right)
\]  
\[\text{(12)}\]
III. PROBABILITY DISTRIBUTION MODEL EVALUATION
In this paper, for the initial evaluation of the process the graphical display of the wind speed of measured data is being superimposed by the fitted PDFs and visual comparison is done. RMSE and $R^2$ criteria are considered for choosing the appropriateness of the fit method for the selected three functions along with their ability to estimate the energy and to predict the wind potential. The performance of these functions is analyzed through the following ways [16]:
A. Root Mean Square Error (RMSE) Test
The value of RMSE that is nearest to the zero indicates a better distribution function. The RMSE is calculated as [17]:
\[
RMSE = \frac{1}{N} \sum_{i=0}^{N} (pi - fi)^2
\]  
\[\text{(13)}\]
B. $R^2$ Test
The R-squared goodness of fit test is hypothesis and comparison testing that determines the correlation between the predicted and observed data. Greater value of $R^2$ shows better fit of the PDF. It can be defined as [18]:
\[
R^2 = 1 - \frac{\sum_{i=1}^{N} (pi - fi)^2}{\sum_{i=1}^{N} (pi^2)}
\]  
\[\text{(14)}\]
C. Wind Power Density Function
It is defined as the power generated by wind turbine per m$^2$. It is helpful in the estimation of wind resources at a site. There are many ways to determine wind power density [6], and it can be defined as follows:
\[
WPD = \sum_{i=1}^{N} \frac{1}{2} \rho v_i^3 f(v_i)
\]  
\[\text{(15)}\]
where the probability of wind speed at the i-th speed value is $f(v_i)$. The standard value of air density is considered to be $\rho=1.225\text{kg/m}^3$.

IV. RESULTS AND DISCUSSION
The proposed site, Tando Ghulam Ali located near Hyderabad, Sindh province, lies in the southern region of Pakistan, at 24°52’02.025''N and 66°51’41.983''E. One year’s wind speed data (from Jan 2017 to Dec 2017), with 10 minute sample intervals were used. The typical hourly and monthly measurements are calculated at the different heights of 80, 60, 40 and 20m. The average wind speed at different heights is shown in Figure 1(a-b). Table I summarizes the average values of wind speed and wind power density for all heights.

![Image](image_url)  
**Fig. 1.** (a) Hourly and (b) monthly average wind speed at different heights.
Table I. Monthly average of wind speed and wind power density

<table>
<thead>
<tr>
<th>Months/Factors</th>
<th>80m</th>
<th>60m</th>
<th>40m</th>
<th>20m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vavg (m/s)</td>
<td>WPD (Watts/m²)</td>
<td>Vavg (m/s)</td>
<td>WPD (Watts/m²)</td>
</tr>
<tr>
<td>Jan</td>
<td>5.948</td>
<td>199.339</td>
<td>5.492</td>
<td>146.349</td>
</tr>
<tr>
<td>Mar</td>
<td>5.967</td>
<td>188.372</td>
<td>5.546</td>
<td>147.580</td>
</tr>
<tr>
<td>Apr</td>
<td>7.999</td>
<td>394.927</td>
<td>7.491</td>
<td>326.331</td>
</tr>
<tr>
<td>June</td>
<td>8.568</td>
<td>485.264</td>
<td>8.206</td>
<td>431.087</td>
</tr>
<tr>
<td>Sep</td>
<td>6.351</td>
<td>181.119</td>
<td>5.908</td>
<td>146.452</td>
</tr>
<tr>
<td>Oct</td>
<td>5.705</td>
<td>166.097</td>
<td>5.171</td>
<td>114.592</td>
</tr>
<tr>
<td>Nov</td>
<td>5.506</td>
<td>176.713</td>
<td>5.082</td>
<td>128.415</td>
</tr>
<tr>
<td>Dec</td>
<td>6.859</td>
<td>271.994</td>
<td>6.141</td>
<td>181.989</td>
</tr>
<tr>
<td>Year</td>
<td>6.990</td>
<td>3.010.112</td>
<td>6.531</td>
<td>243.228</td>
</tr>
</tbody>
</table>

Figure 2 shows the fitting of each PDF over the data. It can be observed that for 80m, 60m, and 40m, the Rayleigh distribution fitted best, followed by gamma, generalized extreme value and lognormal. Due to the lower wind speed at 20m, lognormal fitted best followed by Rayleigh, gamma and generalized extreme value. Table II shows the value of the parameters for each PDF. Table III shows the statistical errors for the different distribution functions and the value of both RMSE and R² goodness of fit tests. For economical evaluation the cost of each turbine is considered as $1.5/W, other initial costs like transportation, installation, grid integration etc. are taken as 40% of the turbine’s cost, the cost of the maintenance and operations is 1.5% of the turbine’s cost at interest rate of 10% and the turbine life span is taken as 20 years.

Table II. Parameters of different PDFs

<table>
<thead>
<tr>
<th>PDF</th>
<th>20m</th>
<th>40m</th>
<th>60m</th>
<th>80m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh</td>
<td>k</td>
<td>c</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>0.00233</td>
<td>-0.1254</td>
<td>-0.09430</td>
<td>-0.2173</td>
</tr>
<tr>
<td>Gamma</td>
<td>α</td>
<td>β</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.443</td>
<td>1.651</td>
<td>1.737</td>
<td>1.835</td>
</tr>
<tr>
<td>GEV</td>
<td>K</td>
<td>A</td>
<td>l</td>
<td>α</td>
</tr>
<tr>
<td></td>
<td>0.506</td>
<td>0.4932</td>
<td>0.5001</td>
<td>0.5172</td>
</tr>
</tbody>
</table>

Table III. Statistical errors at each height

<table>
<thead>
<tr>
<th>Height</th>
<th>Method</th>
<th>RMSE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>80m</td>
<td>Rayleigh</td>
<td>0.01418</td>
<td>0.954265</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.019615</td>
<td>0.840719</td>
</tr>
<tr>
<td></td>
<td>Gen Extreme Value</td>
<td>0.019931</td>
<td>0.834948</td>
</tr>
<tr>
<td>60m</td>
<td>Lognormal</td>
<td>0.026675</td>
<td>0.713073</td>
</tr>
<tr>
<td></td>
<td>Rayleigh</td>
<td>0.010586</td>
<td>0.906878</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.018961</td>
<td>0.875165</td>
</tr>
<tr>
<td></td>
<td>Gen Extreme Value</td>
<td>0.021680</td>
<td>0.841316</td>
</tr>
<tr>
<td>40m</td>
<td>Lognormal</td>
<td>0.02644</td>
<td>0.761952</td>
</tr>
<tr>
<td></td>
<td>Rayleigh</td>
<td>0.010357</td>
<td>0.969184</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.014716</td>
<td>0.937879</td>
</tr>
<tr>
<td></td>
<td>Gen Extreme Value</td>
<td>0.021199</td>
<td>0.879976</td>
</tr>
<tr>
<td>20m</td>
<td>Lognormal</td>
<td>0.00994</td>
<td>0.978188</td>
</tr>
<tr>
<td></td>
<td>Rayleigh</td>
<td>0.010305</td>
<td>0.976476</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.010919</td>
<td>0.972912</td>
</tr>
<tr>
<td></td>
<td>Gen Extreme Value</td>
<td>0.01923</td>
<td>0.921165</td>
</tr>
</tbody>
</table>

Fig. 2. Fitting of different PDFs over the data using MATLAB distribution fitting tool.
V. PERFORMANCE AND COST ASSESSMENTS

The investment cost of a wind power plant is extremely high, it is therefore important to estimate the cost of the energy generated by a turbine in $/kWh, which can be calculated by (16) [19]:

\[ C = \frac{1}{8760I \left( \frac{1}{P_{GR}} \right)} \left[ 1 + m \left( \frac{(1+i)^{-1}}{(1+i)^{-1}} \right) \right] \]

where \( P_b \) is the rated turbine power, \( C_i \) is capacity factor, \( m \) is the cost of operation and maintenance, \( i \) is the rate of interest and \( I \) is the investment cost. To assess the performance and economical evaluation of wind turbines at the candidature site, five different turbines were considered whose specifications are given in Table IV. Each turbine comes with different hub heights. The performance of wind turbines is taken at the hub height of 80m. Estimated capacity factor, annual energy output (kWh/year) and cost/kWh in dollars, for each wind turbine are shown in Table V. The Gamesa G128/4500 wind turbine minimum cost is $0.05453 per kWh of energy generated, with a capacity factor of 40.87%. Nordex N90/2500 model also costs minimum at $0.05453 per kWh with a capacity factor of 32.88%. The highest cost of $0.08019 per kWh energy generated by Vestas V112/3000 with a capacity factor of 27.78%.

![Table IV. Wind Turbines Specifications](image)

![Table V. Estimated Cost per kWh](image)

VI. CONCLUSION

The study concludes that the proposed site can be utilized for commercial purpose. Government of Pakistan (GoP) can exploit this site for wind energy power generation. Four techniques, gamma distribution, Rayleigh distribution, lognormal distribution, and generalized extreme value distribution were employed and their statistical evaluation was conducted by RMSE and R² through Matlab distribution fitting tool. The following points are concluded:

- In comparison to other PDFs, the Rayleigh distribution function fits more accurately to the wind probability distribution at higher altitudes. While at lower wind speeds, lognormal showed better fitting.
- The Gamesa G128 wind turbine model is recommended, having lower cost regarding generated energy, with highest capacity factor of 40.8%.
- More feasibility study of wind power project is required, to exploit wind potential at the proposed site.
- Results showed that the highest wind potential is available during the summer season, during May-July and can be used for power generation.

Calculations were made to obtain the parameter of each PDF, and then the best fitted distribution technique was used to determine the wind potential. This site is in the premises of the National Grid which is another advantage of installing wind projects there. Finally, this site can be employed for commercial purposes.

**ABBREVIATIONS**

- \( v \) = wind speed (m/s),
- \( \rho \) = air density (kg/m³),
- \( \mu \) = mean of wind speed (m/s),
- \( m \) = mean of wind speed (m/s),
- \( f(v) \) = probability density function
- \( s \) = standard deviation
- \( \delta \) = scale parameter of GEV distribution (m/s),
- \( \beta \) = scale parameter of gamma distribution (m/s),
- \( \alpha \) = scale parameter of Rayleigh distribution (dimensionless),
- \( k \) = shape parameter of Rayleigh distribution (dimensionless),
- \( c \) = scale parameter of Rayleigh distribution (m/s),
- \( \gamma \) = shape parameter of GEV distribution (dimensionless),
- \( \phi \) = standard deviation of natural logarithm
- \( I_0 \) = gamma function; \( \lambda \) = mean of natural logarithm
- \( \text{RMSE} \) = root mean square error,
- \( R^2 \) = Coefficient of determination

**REFERENCES**


