Stochastic Optimization of DG Planning in Unbalanced Distribution Networks Under Time-of-Use Pricing

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ABSTRACT

The increasing penetration of Distributed Generation (DG) and demand variability necessitate planning models that explicitly account for uncertainty in Distribution Networks (DNs). This paper proposes a stochastic optimization framework for DG planning in unbalanced networks under time-of-use pricing. The problem is formulated as a bilevel model, where the upper level represents the distribution system operator, and the lower level captures the demand-response behavior. Uncertainty in load and generation is modeled using a scenario-based approach. The original bi-level formulation is reformulated into a single-level problem using Karush-Kuhn-Tucker (KKT) conditions and solved via a metaheuristic optimization method to handle nonlinear and non-convex characteristics. Simulation results on the IEEE 33-bus and 69-bus systems demonstrate consistent improvements in power loss reduction, voltage profile, and operational cost. The findings indicate that these improvements are primarily driven by the integrated modeling framework, which jointly accounts for uncertainty, demand response, and network imbalance, rather than the optimization algorithm itself. The proposed approach provides a robust and practical solution for distribution system planning under uncertainty.

Keywords-stochastic optimization; distributed generation; bi-level optimization; demand response; time-of-use pricing; unbalanced distribution networks

I. INTRODUCTION

The increasing penetration of Distributed Generation (DG), particularly from renewable energy sources, is fundamentally transforming Distribution Networks (DNs) into highly dynamic and uncertain systems [1, 2]. Unlike conventional networks characterized by relatively stable demand, modern distribution systems are driven by time-varying consumption, intermittent generation, and active user participation. These factors introduce significant operational variability, rendering traditional deterministic planning approaches insufficient for accurately capturing real-world system behavior [3, 4].

A critical aspect of this transformation is the growing role of demand response under time-of-use pricing. By shifting electricity consumption in response to price signals, the demand response alters both the magnitude and temporal distribution of the load. In unbalanced DNs, such variations also affect phase loading conditions, thereby influencing voltage profiles and phase imbalance. Despite its importance, demand-side flexibility is often neglected or oversimplified in existing DG planning studies, with limited consideration of its interaction with network constraints [5, 6].

At the same time, uncertainty about load demand and renewable generation further complicates the planning problem. DG planning under such conditions is inherently stochastic, requiring solutions that remain effective across a range of operating scenarios. Although scenario-based approaches have been widely studied, they are typically developed independently of demand response modeling or unbalanced network characteristics, which limits their practical applicability [7, 8].

In addition, the inherently unbalanced nature of DNs introduces further challenges. Unlike transmission systems, DNs commonly operate under asymmetric loading conditions, significantly affecting power losses, voltage deviations, and system reliability. Nevertheless, many existing studies still rely on balanced network assumptions for computational simplicity, potentially leading to suboptimal planning decisions [9].

To address these limitations, this paper proposes a unified stochastic optimization framework for DG planning in unbalanced DNs under time-of-use pricing [6, 10]. The proposed formulation explicitly captures the interaction between DG placement decisions and demand response behavior through a bilevel structure. Uncertainty is incorporated using a scenario-based representation, enabling robust evaluation under varying operating conditions. The resulting bilevel problem was reformulated as a single-level model using Karush-Kuhn-Tucker (KKT) conditions and solved using a population-based optimization approach [8, 11]. The main contributions of this work are as follows: (i) a unified planning framework integrating stochastic uncertainty, TOU-based demand response, and phase imbalance; (ii) a scenario-based formulation for robust performance evaluation; (iii) a bi-level optimization model capturing the interaction between planning and demand response; and (iv) comprehensive validation demonstrating significant improvements in both technical and economic performance.

II. PROBLEM DESCRIPTION

The planning of DG was formulated within the context of unbalanced DNs operating under time-varying demand and price-driven load adjustments. Unlike conventional deterministic approaches, the system was modeled as a dynamic environment in which operating conditions evolve over time and across multiple uncertainty scenarios. Consider a three-phase unbalanced DN with a set of buses N and time intervals T. The load demand at the bus $i \in N$ and time $t \in T$ is expressed as:

$$P_{i,t} = P_{i,t}^{base} + \Delta P_{i,t} \quad (1)$$

where $P_{i,t}^{base}$ denotes the baseline demand and $\Delta P_{i,t}$ represents the adjustment due to demand response low price signals. To ensure consumption feasibility, the total daily energy is preserved:

$$\sum_{t\in\mathcal{T}} P_{i,t}\,\Delta t = E_i \quad (2)$$

where E_i is the daily energy requirement at bus i.

The TOU tariff is defined as a time-dependent electricity price $\lambda(t)$, where the 24-hour operating horizon is divided into off-peak, shoulder, and peak periods, each associated with different price levels. This pricing structure encourages consumers to shift electricity consumption from peak periods to off-peak periods [12]. The electricity cost at each bus can therefore be written as

$$C_i = \sum_{t\in\mathcal{T}} \rho_t \; P_{i,t}\Delta t \quad (3)$$

where p_t denotes the electricity price at time t. The temporal variation induces load redistribution, which in turn affects power flow patterns in the network.

In this study, a typical three-level TOU structure was adopted, where peak prices are higher than off-peak prices to ensure effective demand shifting. DG units can be installed on candidate buses with capacities bounded by:

$$0 \le P_i^{DG} \le P_i^{max} \quad (4)$$

DG units are modeled as renewable-based distributed generators operating at unity power factor. The DG capacity is treated as a continuous decision variable within predefined technical limits, as defined in (4). Candidate buses for DG installation are selected in accordance with typical DN planning practices.

Since DG units are connected across different phases, their placement and sizing influence phase imbalance levels in the network. To capture system uncertainty, a set of scenarios is introduced, where each scenario represents a possible operating condition with an associated probability. The expected value of a given quantity, such as the objective function, is expressed as follows:

$$\mathbb{E}[F] = \sum_{s\in\mathcal{S}} \pi_s\, F^{(s)} \quad (5)$$

This formulation enables the evaluation of planning decisions over a range of operating conditions rather than a single deterministic state.

Based on these elements, the problem was structured as a bi-level optimization model. At the upper level, the Distribution System Operator (DSO) determines DG placement and sizing to improve system performance [13, 14]. At the lower level, consumers adjust their demand to minimize electricity costs under TOU pricing. The interdependence among these levels results in a tightly coupled decision structure in which system states and demand behavior interact dynamically.

III. PROPOSED METHOD

This section presents a bi-level stochastic DG planning model under TOU pricing, reformulated via KKT, and solved using a Marine Predators Algorithm (MPA)-based solution [13, 15].

A. Bi-Level Stochastic Formulation

The DG planning problem was formulated as a bi-level stochastic optimization model. The upper level represents the DSO's decision-making process, while the lower level models demand response under TOU pricing. The upper-level objective was formulated as a normalized weighted aggregation of multiple performance criteria.

$$\min F = w_1 F_{loss} + w_2 F_{volt} + w_3 F_{imb} + w_4 F_{dg} + w_5 F_{cost} \quad (6)$$

subject to:

$$\sum_{k=1}^{5} w_k = 1,\ w_k \geq 0,\ k = 1, \dots, 5$$

where F is the overall objective value and w_k denotes the weighting factor associated with each objective component. All objective components in (6) are normalized with respect to their base case values, ensuring comparability and preventing scale dominance among different indices. This normalization enables a consistent aggregation of multiple performance metrics. The weighting factors were selected to reflect practical engineering priorities in the planning of the DN, where technical criteria such as energy loss, voltage deviation, and phase imbalance were emphasized, while economic factors were also incorporated. Importantly, the proposed formulation is not overly sensitive to the specific choice of weights. It is observed that moderate variations in weighting factors do not significantly affect the optimal DG placement, thereby demonstrating the robustness of the solution.

The superscript base refers to the corresponding reference value obtained from the base case system without DG integration. Each component in (6) represents a normalized performance index and is defined as follows. The expected normalized energy loss is:

$$F_{loss} = \sum_{s \in \mathcal{S}} \pi_s \frac{P_{loss}^{(s)}}{P_{loss}^{base}} \quad (7)$$

where $P_{loss}^{(s)}$ is the total active energy loss under scenario s. The voltage deviation index is:

$$F_{volt} = \sum_{s \in \mathcal{S}} \pi_s \frac{V_{dev}^{(s)}}{V_{dev}^{base}} \quad (8)$$

where $V_{dev}^{(s)}$ denotes the cumulative deviation of nodal voltages from the nominal value. The phase imbalance index is:

$$F_{imb} = \sum_{s \in \mathcal{S}} \pi_s \frac{U_{imb}^{(s)}}{U_{imb}^{base}} \quad (9)$$

where $U_{imb}^{(s)}$ quantifies the degree of phase asymmetry in the DN. The normalized DG investment cost is:

$$F_{dg} = \frac{C_{DG}}{C_{DG}^{base}} \quad (10)$$

where C_{DG} is the total installation cost of DG units. The TOU-based operating cost is:

$$F_{cost} = \sum_{s \in \mathcal{S}} \pi_s \frac{C_{TOU}^{(s)}}{C_{TOU}^{base}} \quad (11)$$

where C_{DG} is the total installation cost of DG units. The TOU-based operating cost is

$$F_{cost} = \sum_{s \in \mathcal{S}} \pi_s \frac{C_{TOU}^{(s)}}{C_{TOU}^{base}} \quad (11)$$

where $C_{TOU}^{(s)}$ is the electricity cost under the scenario s after demand response. The upper-level decision is subject to physical and operational constraints. The DG capacity limits are imposed as follows:

$$0 \leq P_{i,\phi}^{DG} \leq z_{i,\phi} P_{i,\phi}^{max}, \forall i, \phi \quad (12)$$

where $P_{i,\phi}^{DG}$ is the DG capacity at the bus i, phase ϕ, and $z_{i,\phi}$ is a binary installation variable. The maximum number of DG units is limited by:

$$\sum_{i} \sum_{\phi} z_{i,\phi} \leq N_{DG}^{max} \quad (13)$$

The nodal power balance constraint is written as:

$$P_{i,\phi,t}^{grid,(s)} + P_{i,\phi}^{DG} - P_{i,\phi,t}^{load,(s)} = \sum_{j \in \Omega_i} P_{ij,\phi,t}^{(s)} \quad (14)$$

The voltage limits are enforced as:

$$V_i^{min} \leq V_{i,\phi,t}^{(s)} \leq V_i^{max}, \forall i, \phi, t, s \quad (15)$$

The lower-level problem models price-responsive load adjustment:

$$min\ F^L = \sum_{t \in \mathcal{T}} \rho_t \sum_{i} \sum_{\phi} P_{i,\phi,t}^{load} \Delta t \quad (16)$$

subject to:

$$P_{i,\phi,t}^{min} \leq P_{i,\phi,t}^{load} \leq P_{i,\phi,t}^{max} \quad (17)$$

$$\sum_{t \in \mathcal{T}} P_{i,\phi,t}^{load} \Delta t = E_{i,\phi} \quad (18)$$

where ρ_t is the TOU tariff and $E_{i,\phi}$ is the daily energy requirement. The lower-level problem is convex due to its linear objective and affine constraints.

B. KKT-Based Single-Level Reformulation

To eliminate the hierarchical structure, since the lower-level problem is convex, it can be replaced by its KKT optimality conditions. The stationarity condition is given by:

$$\rho_t \Delta t + \lambda_{i,\phi} \Delta t - \mu_{i,\phi,t}^{-} + \mu_{i,\phi,t}^{+} = 0 \quad (19)$$

The primal feasibility conditions are:

$$P_{i,\phi,t}^{min} \leq P_{i,\phi,t}^{load} \leq P_{i,\phi,t}^{max} \quad (20)$$

$$\sum_t P_{i,\phi,t}^{load} \Delta t = E_{i,\phi} \quad (21)$$

The dual feasibility conditions are:

$$\mu_{i,\phi,t}^{-} \geq 0, \mu_{i,\phi,t}^{+} \geq 0 \quad (22)$$

The complementary slackness conditions are

$$\mu_{i,\phi,t}^{-} \left(P_{i,\phi,t}^{min} - P_{i,\phi,t}^{load} \right) = 0 \quad (23)$$

$$\mu_{i,\phi,t}^{+} \left(P_{i,\phi,t}^{load} - P_{i,\phi,t}^{max} \right) = 0 \quad (24)$$

By embedding (19)-(24) into the upper-level problem, the model becomes a single-level nonlinear mixed optimization problem.

C. MPA Solution Framework

The selection of MPA [15] was motivated by the intrinsic characteristics of the formulated optimization problem. Specifically, the problem involved mixed-integer decision variables, nonlinear power flow constraints in unbalanced DNs, and scenario-based stochastic evaluation, resulting in a highly non-convex and complex search space. Conventional metaheuristic methods such as the Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) [16] have been widely applied to similar problems. However, GA may exhibit slow convergence in high-dimensional continuous domains, while PSO is prone to premature convergence when handling constrained and non-convex search spaces.

In contrast, MPA provides a more effective balance between global exploration and local exploitation through its multi-phase search mechanism, enabling improved search capability in complex landscapes. Therefore, MPA is particularly suitable for the proposed problem, where solution quality depends on both global search efficiency and local refinement under mixed-variable and scenario-dependent conditions. MPA is a population-based optimizer for nonlinear constrained problems, where each agent is encoded as a mixed decision vector.

$$X = [z, P^{DG}, P^{load}] \quad (25)$$

where z is the vector of binary DG placement variables, P^{DG} is the vector of DG capacities, and P^{load} is the vector of adjustable load variables over phases and time intervals.

This encoding allows MPA to update both planning and operational variables within a unified search structure. The fitness value assigned to each candidate solution is defined as:

$$Fit(X) = F(X) + \eta \, Pen(X) \quad (26)$$

where $F(X)$ is the objective value computed from (6), η is a positive penalty factor, and $Pen(X)$ is the aggregate constraint violation penalty. The penalty term handles constraint violations, allowing infeasible solutions early in the search. The initial population is generated within bounds.

$$X_i^0 = X^{min} + rand \odot (X^{max} - X^{min}), i = 1, \dots, N_{pop} \quad (27)$$

where X_i^0 is the initial position of the agent i, X^{min} and X^{max} are lower and upper bound vectors, $rand$ is a random vector with entries in $[0, 1]$, $\odot$ denotes element-wise multiplication, and N_{pop} is the population size.

Binary variables are obtained by thresholding, and MPA updates the population through exploration–exploitation phases as follows:

$$X_i^{g+1} = X_i^g + R \odot \left(Elite^g - R \odot X_i^g \right) \quad (28)$$

where X_i^g is the position of the agent I at iteration g, and $Elite^g$ is the best solution so far. R is a random vector.

In the second stage, the search balances exploration and exploitation through Lévy-based motion:

$$X_i^{g+1} = Elite^g + CF^g \odot Levy \odot \left(Elite^g - X_i^g \right) \quad (29)$$

where CF^g is an adaptive control factor at iteration g, and $Levy$ is a vector generated from a Lévy distribution.

In the third stage, exploitation dominates, and Brownian-like movements refine the best region found so far:

$$X_i^{g+1} = Elite^g + CF^g \odot Brownian \odot \left(Elite^g - X_i^g \right) \quad (30)$$

where $Brownian$ is a random vector generated according to Brownian motion.

Equations (28) to (30) follow standard MPA logic. Due to mixed variables and constraints, solutions are repaired before evaluation as follows:

1. DG capacity variables that exceed their bounds are clipped to the nearest feasible limit.
2. Binary DG placement variables are rounded to $\{0,1\}$.
3. Adjustable load variables are rescaled, when needed, to satisfy daily energy conservation in (25).
4. Remaining violations are penalized through $Pen(X)$ in (21).
5. This hybrid repair penalty mechanism improves numerical stability and avoids excessive infeasibility propagation during the search.

The elite solution is updated in each iteration as:

$$Elite^{g+1} = arg \min_{1 \leq i \leq N_{pop}} \left\{ Fit(X_i^{g+1}) \right\} \quad (31)$$

The algorithm terminates when either the maximum number of iterations is reached or the improvement in the elite solution becomes smaller than a prescribed tolerance.

To enhance clarity and reproducibility, the overall solution procedure of the proposed method is summarized step-by-step as follows:

```
Algorithm 1: Proposed Method
Step 1: Read the network data, TOU tariff
  data, scenario information, and MPA
  control parameters.
Step 2: Initialize the population X_i^(0)
  within the feasible bounds of the
  decision variables.
Step 3: Decode each candidate solution
  into DG placement, DG sizing, and load
  adjustment variables.
Step 4: For each scenario, perform an
  unbalanced power flow analysis and
  compute the objective components and
  corresponding fitness values.
Step 5: Update the population X_i^(g) using
  the three search phases of the MPA.
Step 6: Repair infeasible solutions and
  update the global best (elite) solution.
Step 7: Repeat Steps 4-6 until the
  stopping criterion is satisfied.
Step 8: Output the optimal DG placement,
  sizing, and demand response strategy.
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This procedure provides a systematic implementation framework for solving the proposed stochastic DG planning problem under time-of-use pricing. The MPA-based solution approach is well-suited to this problem due to its ability to effectively handle mixed discrete–continuous decision variables, nonlinear and non-convex operational constraints, and scenario-dependent evaluations, without requiring gradient information.

IV. TEST RESULTS

The effectiveness of the proposed method was evaluated on the 33-bus and 69-bus unbalanced distribution systems under 24-hour TOU load conditions. System uncertainty was incorporated through a scenario-based modeling approach that accounts for variations in both load demand and distributed generation output. A set of candidate locations and capacities for DG units was predefined to ensure a consistent and comparable planning framework. The parameters of the MPA were carefully tuned to achieve reliable convergence for the formulated optimization problem.

To enable a comprehensive assessment, four scenarios were considered, including the base case, DG-only integration, DG with TOU-based demand response, and the proposed integrated framework. This configuration allows a systematic evaluation of the individual and combined impacts of DG deployment and demand response strategies. The optimal DG placement and sizing results for the 33-bus and 69-bus systems are summarized in Tables I and III, respectively.

A. The 33-bus System

The 33-bus distribution system was adopted as a benchmark test network, operating at a nominal voltage level of 12.66 kV. The system consists of 33 buses and 37 branches, representing a typical radial distribution structure. The total load demand of the system was (3.72 + j2.3) MVA. Figure 1 illustrates the single-line diagram of the network. This test system has been widely used in the literature to evaluate the effectiveness of planning and optimization strategies in DNs [17].

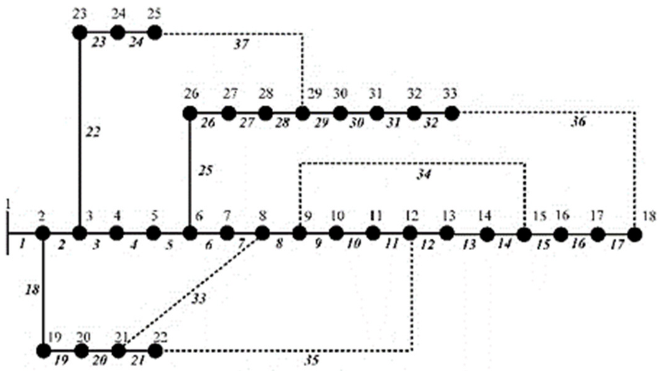

Fig. 1. Diagram of the 33-bus DPS.

As shown in Table I, the integration of DG significantly reduces the loss index F_{loss} from 1.000 to 0.773, corresponding to a 22.7% reduction compared to the base case. When TOU-based demand response is incorporated, the loss index further decreases to 0.731, achieving an additional improvement of 4.2%. The proposed method yields the lowest value of 0.644, corresponding to a total reduction of 35.6%.

In terms of voltage performance, the deviation index F_{volt} is reduced to 0.574, representing a 42.6% improvement, while the phase imbalance index F_{imb} decreases to 0.620. These results confirm the effectiveness of phase-level optimization in addressing unbalanced operating conditions. Furthermore, the TOU-based cost index F_{cost} is reduced by 16.3%, demonstrating notable economic benefits.

TABLE I. PERFORMANCE COMPARISON ON THE 33-BUS SYSTEM

Case	DG (bus)	P^{DG} (kW)	F_{loss}	F_{volt}	F_{imb}	F_{cost}
Base	–	–	1	1	1	1
DG only	6, 17, 30	920, 650, 410	0.773	0.742	0.817	0.938
DG+TOU	6, 17, 31	950, 670, 430	0.731	0.674	0.732	0.891
Proposed	5, 16, 31	980, 710, 450	0.644	0.574	0.62	0.837

The results in Table II further validate the superiority of the proposed method over conventional optimization algorithms. Compared with GA, the loss index is reduced from 0.712 to 0.644, corresponding to a 9.6% improvement, while PSO achieves a smaller gain of 5.8%. Although the standard MPA already provides competitive performance with a loss index of 0.662, the proposed framework further improves this value by 2.7%. It should be noted that the proposed case corresponds to the application of MPA within the integrated modeling framework, whereas the MPA case represents the standard use of MPA without modeling enhancements. This distinction highlights that the observed performance gains are primarily attributed to the proposed formulation rather than the optimization algorithm itself.

TABLE II. ALGORITHM COMPARISON ON THE 33-BUS SYSTEM

Method	F_{loss}	F_{volt}	F_{imb}	F_{cost}
GA	0.712	0.689	0.741	0.873
PSO	0.684	0.655	0.703	0.859
MPA	0.662	0.621	0.671	0.846
Proposed	0.644	0.574	0.62	0.837

From a system perspective, the improvements can be explained by the coordinated effects of DG placement, TOU-based load shifting, and phase-level optimization. DG integration reduces power transfer distances and associated losses, while TOU mechanisms redistribute load away from peak periods, alleviating system stress. Meanwhile, phase-level optimization mitigates imbalance in unbalanced networks. Overall, the results demonstrate that the proposed approach not only improves individual objective indices but also enhances overall system performance consistently and robustly.

B. The 69-Bus System

The 69-bus distribution system was employed as a larger-scale benchmark to further validate the effectiveness of the proposed framework. The system operates at a nominal voltage level of 12.66 kV and consists of a detailed radial configuration of buses and branches, with system parameters adopted from [17]. In the absence of DG integration, the network exhibits a relatively high power loss of 224.88 kW, indicating significant potential for improvement using optimal planning strategies. Figure 2 illustrates the single-line diagram of the system, providing a comprehensive representation of the network topology and interconnections. This test system offers a more complex and realistic operating environment compared to the 33-bus case, thereby enabling a rigorous assessment of the proposed method under increased network scale and operational complexity.

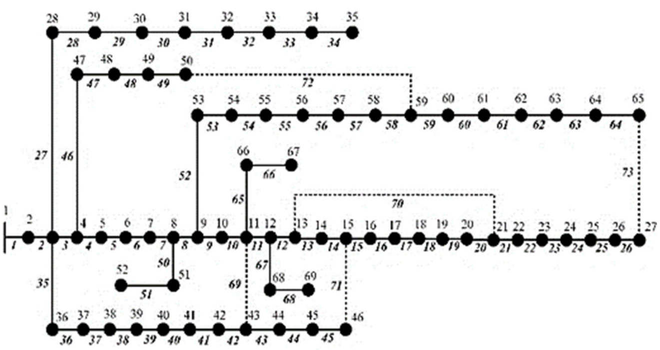

Fig. 2. Diagram of the 69-bus DPS

The results in Table III confirm the effectiveness and scalability of the proposed framework under more complex operating conditions. The integration of DG reduces the loss index to 0.801, corresponding to a 19.9% reduction compared to the base case. When TOU-based demand response is incorporated, the loss index is further reduced to 0.748, achieving an additional improvement of 5.3%. The proposed method attains the lowest value of 0.682, resulting in a total reduction of 31.8%. Regarding voltage performance, the deviation index decreases to 0.611, corresponding to an improvement of 38.9%, while the phase imbalance index is reduced to 0.648, indicating enhanced system balance. In addition, the TOU-based cost index decreases by 14.9%, demonstrating consistent economic benefits.

TABLE III. PERFORMANCE COMPARISON ON THE 69-BUS SYSTEM

Case	DG (bus)	P^{DG} (kW)	F_{loss}	F_{volt}	F_{imb}	F_{cost}
Base	–	–	1	1	1	1
DG Only	13, 25, 61	980, 820, 910	0.801	0.786	0.842	0.951
DG + TOU	13, 26, 63	1000, 850, 950	0.748	0.701	0.763	0.902
Proposed	15, 27, 64	1000, 870, 980	0.682	0.611	0.648	0.851

Compared to the 33-bus system, the performance gains are slightly lower, which can be attributed to the increased network complexity and reduced load flexibility. Nevertheless, the proposed method maintains consistent effectiveness, confirming its scalability to larger distribution systems.

The comparative results shown in Table IV further demonstrate the superiority of the proposed method over conventional optimization algorithms. Compared to GA, the loss index is reduced from 0.745 to 0.682, corresponding to an 8.5% improvement, while PSO achieves a smaller improvement of 5.4%. Although the standard MPA provides competitive performance with a loss index of 0.701, the proposed framework further improves it by 2.7%. It should be noted that the proposed case corresponds to the application of MPA within the integrated modeling framework, whereas the MPA case represents the standard use of MPA without modeling enhancements. This distinction confirms that the observed performance gains are primarily attributed to the proposed formulation rather than the optimization algorithm itself.

TABLE IV. ALGORITHM COMPARISON ON THE 69-BUS SYSTEM

Method	F_{loss}	F_{volt}	F_{imb}	F_{cost}
GA	0.745	0.722	0.781	0.901
PSO	0.721	0.698	0.742	0.884
MPA	0.701	0.662	0.702	0.869
Proposed	0.682	0.611	0.648	0.851

From a system-level perspective, these improvements can be explained by the coordinated effects of DG placement, TOU-based load shifting, and phase-level optimization. DG integration reduces power transfer distances and associated losses, while TOU mechanisms alleviate peak demand and improve voltage profiles. Meanwhile, phase-level optimization effectively mitigates imbalance in unbalanced networks. Overall, the results demonstrate that the proposed framework achieves consistent performance improvements and maintains robustness under increased system scale and complexity.

V. CONCLUSIONS

This paper presents a stochastic bi-level optimization framework for DG planning in unbalanced DNs under TOU pricing. The proposed formulation captures the interaction between network planning decisions and demand response, while explicitly incorporating uncertainty through a scenario-based approach. The original bi-level problem was reformulated into a single-level model using KKT conditions and solved via a metaheuristic optimization method. The simulation results on the 33-bus and 69-bus systems demonstrate consistent improvements in both technical and economic performance, including reductions in energy losses, voltage deviations, phase imbalance, and operating costs. These improvements are achieved through coordinated modeling of DG placement, TOU-based demand response, and unbalanced network characteristics. Importantly, the results indicate that the performance gains are primarily driven by the integrated modeling framework rather than the optimization algorithm itself. This highlights that accurate problem formulation, including the representation of uncertainty and system characteristics, plays a more critical role than the choice of solution method.

Overall, the proposed framework provides a robust and practically applicable approach for distribution system planning under uncertainty and can be readily extended to other distribution systems with similar characteristics. Future work will focus on enhanced renewable uncertainty modeling, dynamic network reconfiguration, and multi-energy system integration, along with further investigation of weighting factor sensitivity.

DECLARATION OF COMPETING INTERESTS

The authors declare no conflict of interest.

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Not applicable to this work.

DATA AVAILABILITY

The data used in this study are available from the corresponding author upon reasonable request.

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