

# AW: An Aggregated Weighting Method for Balancing Expert Opinions

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## ABSTRACT

Determining criteria weights is significant in multi-objective optimization, particularly in Multi-Criteria Decision-Making (MCDM) problems. This study introduces a novel criteria weighting approach, the Aggregated Weighting (AW) method, which integrates three widely used subjective weighting techniques: Rank Reciprocal (RR), Rank Sum (RS), and Rank Order Centroid (ROC). Its effectiveness is demonstrated through three case studies, including two synthetic examples and one real-world application involving the evaluation of wood milling machines. The results show that the AW method consistently produces rankings that reflect a balanced consensus compared to those obtained using the individual component methods (RR, RS, and ROC). Therefore, the AW method offers a practical and reliable framework for supporting group decision-making when different preferences exist regarding the choice of weighting technique.

*Keywords-AW method; weight method; MCDM*

## I. INTRODUCTION

Multi-Criteria Decision-Making (MCDM) is used to rank alternatives in order to identify the most suitable option and eliminate inferior ones, with applications spanning numerous fields [1]. Within MCDM, the final ranking of alternatives is strongly influenced by the method used to determine criteria weights [2]. These weights represent the relative importance of each criterion and can be derived using either objective or subjective approaches [3]. A variety of objective weighting methods have been developed, including Entropy [4], LOPCOW [5], SPC [6], SD [7], MEREC [8], CRITIC [9], LOGSTA [10], SITDE [11], CRISUS [12], and LODECI [13].

Despite their widespread use, objective methods have a key limitation: they do not account for the decision-maker's preferences regarding the importance of criteria, which may lead to less satisfactory decision outcomes [14]. Moreover, different objective methods can yield inconsistent or even conflicting weight values, where the same criterion may receive a high weight under one method and a low weight under another [15].

Conversely, subjective weighting methods calculate weights based on user opinions regarding importance, ensuring the ranking aligns with expectations [16]. Subjective weighting methods were evaluated in [17]. However, the existence of

multiple subjective methods poses a challenge in reaching consensus, as different users may prefer different methods, with each method generating distinct weight values, thus affecting the final ranking [18]. The PIPRECIA subjective weighting method evaluates the relative importance of criteria through pairwise comparisons against the preceding criterion, thereby calculating intermediate coefficients to determine the final weights [19]. However, the final results depend heavily on the selection and evaluation of the initial reference criterion. If the decision-maker lacks a comprehensive overview from the outset, the weight structure may misrepresent objective reality. The AHP (subjective) weighting method performs pairwise comparisons to assess the relative importance between elements, from which criteria weights are calculated [20]. Nevertheless, this method has a limitation: as the number of criteria increases, the pairwise comparison matrix becomes overly complex, making it difficult to ensure consistency and computational efficiency. Additionally, this method requires high implementation costs and is prone to conflicts between expert opinions due to the inherent subjectivity of the evaluations. The FUCOM subjective weighting method also ranks criteria by importance, followed by pairwise comparisons between adjacent criteria in the ranking to determine relative priority coefficients [21]. Yet, this method is limited by the fact that the results depend significantly on the expert's subjectivity during the initial ranking and evaluation steps. Furthermore, solving the non-linear optimization model to find the weights may require specialized computational software. The LMAW subjective weighting method employs a logarithmic scale to process expert opinions [22]. A drawback of this method is the relatively complex calculation process involving multiple intermediate steps and the requirement to establish stability parameters. Moreover, selecting an appropriate logarithmic base still depends on the researcher's subjective decision, which can affect the results if not correctly configured.

The application of many weighting methods is often hindered by computational complexity and the increasing risk of inconsistency as the number of criteria grows. In addition, their implementation may require specialized software to solve optimization models or manage complex intermediate steps. These limitations pose challenges, particularly in time-sensitive decision-making contexts or for practitioners with limited expertise in mathematics and programming. Among subjective approaches, three widely used methods in MCDM are Rank Reciprocal (RR), Rank Sum (RS), and Rank Order Centroid (ROC). A significant advantage shared by these methods is their simplicity, as each relies on a single formula to compute criteria weights based solely on the ranking order of criteria [23]. However, due to differences in their underlying formulations, these methods can produce significantly different weight distributions for the same set of criteria [23]. This creates a practical challenge: when multiple stakeholders adopt different methods (e.g., RR, RS, or ROC), achieving consensus on the final ranking becomes difficult. To address this issue, this study proposes a synthesized weighting approach, namely the AW method, which integrates RR, RS, and ROC. This approach aligns with the view that relying on a single weighting method may not adequately capture the true importance of criteria, as no method has been proven

universally optimal [24]. Instead, combining multiple methods is expected to improve the robustness and reliability of decision outcomes [24]. Rather than positioning AW as a superior alternative, this study presents it as a consensus-oriented compromise solution. It is particularly suited to decision-making contexts in which experts agree on the ranking of criteria but differ in their preferred ordinal weighting technique (RR, RS, or ROC).

## II. THE AW METHOD

Assume that surveyed experts consistently identify the priority order of criteria but disagree on the subjective weighting method to apply. Suppose among the methods surveyed, some select RS, some RR, and others ROC. AW is designed to aggregate these three methods. Therefore, the formulas for the component methods must be first presented.

Let  $n$  be the number of criteria and  $k$  be the priority rank of criterion  $k$ . The weights for criterion  $k$  according to RR, RS, and ROC are presented in [23]:

$$W_{k,RR} = \frac{2(n+1-k)}{n(n+1)} \quad (1)$$

$$W_{k,RS} = \frac{\frac{1}{k}}{\sum_{j=1}^n \frac{1}{j}} \quad (2)$$

$$W_{k,ROC} = \frac{1}{n} \sum_{k=1}^n \frac{1}{k} \quad (3)$$

After calculating the criteria weights using the RR, RS, and ROC methods, the next step is to aggregate these values into a single set of final weights through the proposed AW method. A common aggregation approach assigns priority coefficients to each component method, giving higher weights to the preferred methods and lower weights to the less preferred ones [25–27]. However, this approach is inherently subjective, as the selection of coefficients depends on individual preferences, potentially introducing bias and undermining fairness among the methods. To address this limitation, the present study adopts the geometric mean, as defined in (4), to combine the weights obtained from the three methods. This aggregation technique treats all component methods equally, thereby ensuring a fair and unbiased integration, particularly in situations where decision-makers rely on different weighting approaches.

$$V_{k,AW} = \sqrt[3]{W_{k,RR} \cdot W_{k,RS} \cdot W_{k,ROC}} \quad (4)$$

The weight of criterion  $k$  represents the weight determined by the AW method and is calculated using:

$$W_{k,AW} = \frac{V_{k,AW}}{\sum_{k=1}^n V_{k,AW}} \quad (5)$$

## III. EVALUATING THE EFFECTIVENESS OF THE AW METHOD

### A. Illustrative Example 1

Consider a decision-making problem involving the ranking of 14 alternatives, denoted as A1-A14. Each alternative is evaluated based on six criteria (C1–C6), of which the first four are benefit criteria (Type B, where higher values are preferred) and the remaining two are cost criteria (Type C, where lower

values are preferred). The dataset used in this example is presented in Table I.

TABLE I. DATA FOR EXAMPLE 1

Alt.	C1	C2	C3	C4	C5	C6
A1	7.6	46	18	390	0.1	11
A2	5.5	32	21	360	0.05	11
A3	5.3	32	21	290	0.05	11
A4	5.7	37	19	270	0.05	9
A5	4.2	38	19	240	0.1	8
A6	4.4	38	19	260	0.1	8
A7	3.9	42	16	270	0.1	5
A8	7.9	44	20	400	0.05	6
A9	8.1	44	20	380	0.05	6
A10	4.5	46	18	320	0.1	7
A11	5.7	48	20	320	0.05	11
A12	5.2	48	20	310	0.05	11
A13	7.1	49	19	280	0.1	12
A14	6.9	50	16	250	0.05	10

Suppose all experts consistently prioritize the criteria in the following order: C1 > C2 > C3 > C4 > C5 > C6. This represents a specific scenario, designated as Scenario S1. Further scenarios will be investigated in the subsequent sensitivity analysis section of this paper. The calculation of weights for each criterion is performed as:

- For the RR method, applying (1), the weight of criterion C1 is  $2 \times (6+1-1) / (6 \times (6+1)) = 0.4082$ , and the weight of criterion C2 is  $2 \times (6+1-2) / (6 \times (6+1)) = 0.2041$ . Following the same procedure, the weights for criteria C3, C4, C5, and C6 are 0.1361, 0.1020, 0.0816, and 0.0680, respectively.
- For the RS method, applying (2), the weight of criterion C1 is  $(1/1) / (1/1+1/2+1/3+1/4+1/5+1/6) = 0.2857$ , and the weight of criterion C2 is  $(1/2) / (1/1+1/2+1/3+1/4+1/5+1/6) = 0.2381$ . Similarly, the weights for criteria C3, C4, C5, and C6 are 0.1905, 0.1429, 0.0952, and 0.0476, respectively.
- For the ROC method, applying (3), the weight of criterion C1 is  $(1/6) \times (1/1+1/2+1/3+1/4+1/5+1/6) = 0.4083$ , and the weight of criterion C2 is  $(1/6) \times (1/2+1/3+1/4+1/5+1/6) = 0.2417$ . Likewise, the weights for criteria C3, C4, C5, and C6 are 0.1583, 0.1028, 0.0611, and 0.0278, respectively.
- Applying (4), the geometric mean for C1 is calculated as  $(0.4082 \times 0.2857 \times 0.4083)^{1/3} = 0.3625$ . By following the same steps, the geometric mean values for criteria C2-C6 are determined as 0.2273, 0.1601, 0.1144, 0.0780, and 0.0448, respectively.
- Applying (5), the weight of C1 using the AW method is  $0.3625 / (0.3625 + 0.2273 + 0.1601 + 0.1144 + 0.0780 + 0.0448) = 0.3672$ . The weights for the remaining criteria from C2 to C6, using the AW method, are calculated similarly. All calculated results are summarized in Table II.

TABLE II. CRITERIA WEIGHTS IN EXAMPLE 1

Weight method	C1	C2	C3	C4	C5	C6	max/min
RR	0.4082	0.2041	0.1361	0.1020	0.0816	0.0680	6
RS	0.2857	0.2381	0.1905	0.1429	0.0952	0.0476	6
ROC	0.4083	0.2417	0.1583	0.1028	0.0611	0.0278	14.7
AW	0.3672	0.2303	0.1622	0.1159	0.079	0.0454	8.09

Based on Table II, the ratio between the maximum weight (C1) and the minimum weight (C6) for both RS and RR methods is identical and equal to the number of criteria (i.e., 6). In contrast, this ratio reaches its peak with the ROC method, while the AW method yields an intermediate value. This variance stems from the distinct weight distribution logic inherent in the mathematical formulations of each method. However, these discrepancies raise a significant question: Is the AW method truly effective in weight determination? A weighting method is considered effective if it maintains high rank stability for alternatives when integrated into various MCDM frameworks. To validate the utility of the AW method, this study employs five MCDM techniques to rank the alternatives: FUCA, SAW, PIV, ROV, and RAM. Figures 1-5 present the rankings of the alternatives using FUCA, SAW, PIV, ROV, and RAM, respectively. Rankings are generated using weights derived from four methods: AW, RR, RS, and ROC.

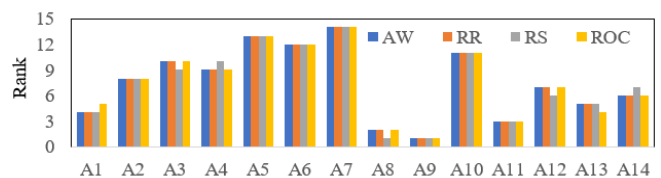


Fig. 1. Ranking of alternatives in example 1 using the FUCA method.

The results of the FUCA method are illustrated in Figure 1. It is observed that alternatives A2, A6, A7, A8, A9, A10, and A11 maintain perfectly consistent rankings regardless of which weighting method is applied. Other alternatives exhibit negligible fluctuations. For instance, A1 consistently ranks 4<sup>th</sup> under the AW, RR, and ROC weights, only shifting to 5<sup>th</sup> when ROC is applied (Note: if there is a discrepancy in your original text regarding ROC/RS, please double-check; here I translated as written). Similarly, A3 holds the 10<sup>th</sup> position under AW, RR, and ROC, while placing 9<sup>th</sup> under the RS method. The remaining alternatives also show high stability, typically maintaining the same rank in 3 out of 4 cases with only minor deviations in the last case. Consequently, preliminary findings suggest that when utilizing FUCA, the AW method yields results that are highly compatible with the RR, RS, and ROC methods.

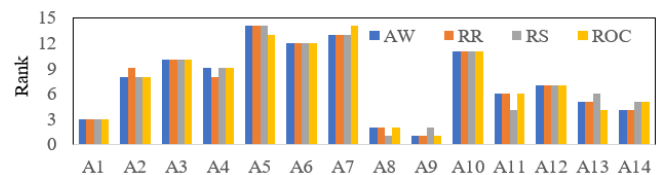


Fig. 2. Ranking of alternatives in example 1 using the SAW method.

In the case of the SAW method, as depicted in Figure 2, alternatives A1, A3, A6, A10, and A12 demonstrate total rank consistency across all four weighting scenarios. Other alternatives show minimal shifts; for example, A2 ranks 8<sup>th</sup> using AW, RS, and ROC, but drops to 9<sup>th</sup> with RR weights. Other alternatives follow a similar pattern of stability in at least

3 out of 4 instances. Thus, it can be initially inferred that the AW method produces alternative rankings that are highly compatible with RR, RS, and ROC when paired with the SAW ranking algorithm.

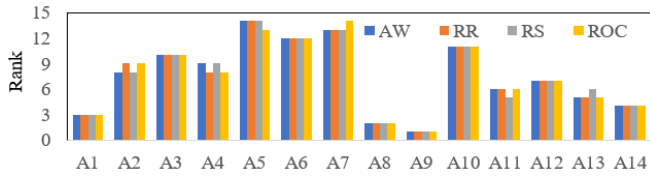


Fig. 3. Ranking of alternatives in example 1 using the PIV method.

The PIV ranking results, as shown in Figure 3, also reveal consistent positioning for several alternatives, specifically A1, A3, A6, A8, A9, A10, A12, and A14. The remaining alternatives demonstrate high stability with only slight variations between weight sets. This alignment further supports the premise that the AW method produces results that are highly compatible with RR, RS, and ROC, effectively serving as a balanced compromise among these ordinal weighting techniques within the PIV framework.

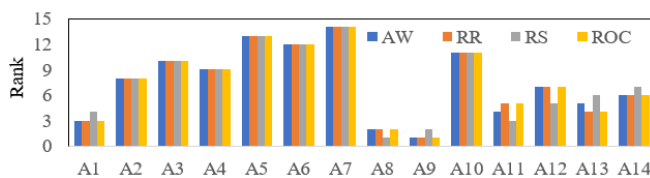


Fig. 4. Ranking of alternatives in example 1 using the ROV method.

For the ROV method, as portrayed in Figure 4, a significant number of alternatives, including A2, A3, A4, A5, A6, A7, and A10, maintain identical ranks across all four weighting approaches. Any shifts observed in other alternatives are marginal, with consistency maintained in most test cases. This demonstrates that the AW method produces alternative rankings that are highly compatible with those generated by RR, RS, and ROC under the ROV model.

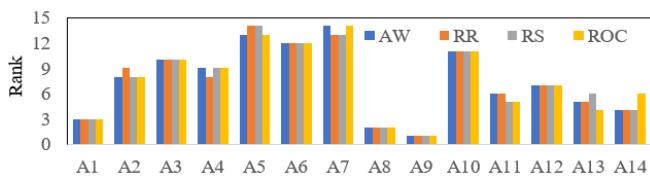


Fig. 5. Ranking of alternatives in example 1 using the RAM method.

When applying the RAM method, as depicted in Figure 5, alternatives A1, A3, A6, A8, A9, A10, and A12 consistently maintain their ranks across all four scenarios, with only minimal fluctuations observed for the remaining alternatives. From a preliminary qualitative perspective, the AW method appears to foster a balanced consensus with the other three methods. Overall, across all five MCDM frameworks (FUCA, SAW, PIV, ROV, and RAM), the AW weighting method performs consistently alongside RR, RS, and ROC. A cross-analysis of Figures 1–5 further shows that A8 and A9 emerge

as the most favorable alternatives, whereas A5, A6, A7, and A11 are consistently the least favorable, regardless of the ranking or weighting method used. This consensus indicates that the AW method provides a balanced and reliable compromise, effectively capturing the main trends of the individual weighting approaches. However, since these observations are qualitative, a rigorous quantitative evaluation is necessary. In this study, the Spearman rank correlation coefficient is employed to assess the performance of the AW method relative to the others [28]. Using the data from Figures 1–5, Spearman coefficients between the AW and the RR, RS, and ROC methods were calculated for each MCDM framework, and are summarized in Table III.

TABLE III. SPEARMAN CORRELATION COEFFICIENTS BETWEEN AW AND OTHER WEIGHTING METHODS IN EXAMPLE 1

MCDM method	Weight method			Average
	RR	RS	ROC	
FUCA	0.9956	0.9824	0.9912	0.9897
SAW	0.9956	0.9956	0.9912	0.9941
PIV	0.9956	0.9956	0.9912	0.9941
ROV	0.9956	0.9780	0.9956	0.9897
RAM	0.9912	0.9912	0.9868	0.9897
Average				0.9915

The analysis of the FUCA results, as displayed in Figure 1, indicates that alternatives A2, A6, A7, A8, A9, A10, and A11 exhibit complete ranking consistency across all weighting methods. The remaining alternatives show only minor variations. For example, A1 ranks 4th under the AW, RR, and RS methods, and shifts slightly to 5th under the ROC method. Similarly, A3 is ranked 10th under AW, RR, and ROC, and improves marginally to 9th under RS. Overall, most alternatives maintain identical rankings in at least three out of the four weighting scenarios, with only negligible deviations in the remaining case. These observations suggest that, when applied within the FUCA framework, the AW method produces rankings that are highly consistent with those obtained using the RR, RS, and ROC methods.

B. Illustrative Example 2

In this scenario, the AW method is evaluated against the RR, RS, and ROC methods within a decision-making problem, involving the ranking of 10 alternatives, each characterized by 12 distinct criteria. The alternatives are designated as A1-A10, and the criteria are labeled as C1-C12. Specifically, the first seven criteria are categorized as benefit-type (Type B), while the remaining five are cost-type (Type C). The simulated dataset for Example 2 is provided in Table IV.

TABLE IV. DATASET FOR EXAMPLE 2

Alt.	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
A1	85	12.5	0.82	450	9	120	5.5	0.91	32	1500	0.45	88
A2	72	14.2	0.75	520	6	110	4.2	0.85	28	1750	0.52	76
A3	94	10.8	0.9	380	6	145	6.8	0.95	40	1200	0.4	92
A4	68	15.5	0.68	600	5	95	3.5	0.8	25	2000	0.6	65
A5	80	13	0.8	480	8	130	5	0.88	35	1400	0.48	82
A6	88	18	0.85	420	7	125	6.2	0.92	38	1350	0.42	50
A7	75	13.8	0.78	500	6	115	4.8	0.86	30	1600	0.38	78
A8	91	11.2	0.88	400	9	140	7.2	0.94	42	1100	0.4	95
A9	65	16	0.65	650	4	90	3.2	0.78	22	2100	0.65	60
A10	82	12.8	0.81	460	8	122	5.2	0.89	33	1450	0.47	84

Assume that the priority order of the criteria is established as  $C1 > C2 > C3 > C4 > C5 > C6 > C7 > C8 > C9 > C10 > C11 > C12$ . Following the procedural steps described in Example 1, the criteria weights were calculated and are summarized in Table V. Consistent with the previous findings, the ratio between the maximum weight (C1) and the minimum weight (C12) is equal to the total number of criteria (12) for both the RS and RR methods. In contrast, the ROC method

shows the highest weight ratio, while the AW method falls at an intermediate level between these extremes. Using the same analytical approach as in Example 1, Spearman rank correlation coefficients were calculated to compare the AW method with the RR, RS, and ROC methods across five different MCDM ranking frameworks. The results are outlined in Table VI.

TABLE V. CRITERIA WEIGHTS FOR EXAMPLE 2

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	max/min
RR	0.3222	0.1611	0.1074	0.0806	0.0644	0.0537	0.046	0.0403	0.0358	0.0322	0.0293	0.0269	12
RS	0.1538	0.141	0.1282	0.1154	0.1026	0.0897	0.0769	0.0641	0.0513	0.0385	0.0256	0.0128	12
ROC	0.2586	0.1753	0.1336	0.1058	0.085	0.0683	0.0544	0.0425	0.0321	0.0229	0.0145	0.0069	37.24
AW	0.2396	0.1623	0.1255	0.1018	0.0845	0.0707	0.0591	0.049	0.0398	0.0312	0.0227	0.0137	17.5

TABLE VI. SPEARMAN CORRELATION COEFFICIENTS BETWEEN AW AND OTHER WEIGHTING METHODS IN EXAMPLE 2

MCDM method	Weight method			Average
	RR	RS	ROC	
FUCA	0.9879	1	0.9879	0.9919
SAW	0.9879	0.9879	1	0.9919
PIV	0.9758	0.9879	1	0.9879
ROV	1	0.9879	1	0.9960
RAM	0.9758	1	1	0.9919
Average				0.9919

The results of this example further confirm that the Spearman correlation coefficients between the AW method and the RR, RS, and ROC methods remain consistently high across all five MCDM techniques. The overall average Spearman coefficient is 0.9919, which is very close to 1 and well above the 0.8 threshold for strong agreement [29]. These findings indicate that using the AW method for criteria weighting yields alternative rankings that are highly consistent with those obtained from the RR, RS, and ROC methods. Therefore, the AW method effectively reconciles differing methodological preferences and promotes a high level of consensus among decision-makers in determining criteria weights.

C. Illustrative Example 3

The two preceding synthetic examples, despite variations in the number of criteria, criterion types, and the quantity of alternatives, consistently demonstrated that the AW method achieves an efficacy comparable to the RR, RS, and ROC

methods in determining criterion weights. To further substantiate this finding, these three examples address a real-world problem: weight determination for ranking four types of wood milling machines (denoted A1-A4) based on data from a recently published study. Each machine is evaluated against eight benefit-type criteria (C1-C8) and one cost-type criterion (C9). These nine criteria encompass: X-axis working range (C1), Y-axis working range (C2), Z-axis working range (C3), maximum spindle speed (C4), maximum travel speed (C5), maximum working speed (C6), maximum permissible wood moisture content (C7), flash memory (C8), and price in Vietnamese currency (C9). The dataset for this practical application is consolidated in Table VII, with units for each criterion specified in the second row [30].

TABLE VII. DATASET FOR EXAMPLE 3

Alt.	C1	C2	C3	C4	C5	C6	C7	C8	C9
	mm	mm	mm	mm/min	mm/min	mm/min	°C	Mb	million
A1	1800	2500	180	24000	50	25	75	128	164
A2	1500	1300	180	23000	42	22	70	100	149
A3	1500	1000	150	20000	50	25	50	120	129
A4	1800	2000	160	21500	50	25	50	120	154.56

Assuming that the priority hierarchy of the criteria remains  $C1 > C2 > C3 > C4 > C5 > C6 > C7 > C8 > C9$ . As established, this is a single test case, and further scenarios will be detailed in the subsequent sensitivity analysis. Table VIII synthesizes the criteria weights calculated using the various methodologies.

TABLE VIII. CRITERIA WEIGHTS FOR EXAMPLE 3

	C1	C2	C3	C4	C5	C6	C7	C8	C9	max/min
RR	0.3535	0.1767	0.1178	0.0884	0.0707	0.0589	0.0505	0.0442	0.0393	9
RS	0.2	0.1778	0.1556	0.1333	0.1111	0.0889	0.0667	0.0444	0.0222	9
ROC	0.3143	0.2032	0.1477	0.1106	0.0828	0.0606	0.0421	0.0262	0.0123	25.4607
AW	0.2864	0.189	0.142	0.1113	0.0883	0.0695	0.0531	0.0379	0.0225	12.7287

In this instance, it is once again observed that the ratio between the maximum weight (C1) and the minimum weight (C9) for both the RS and RR methods is identical to the total number of criteria (i.e., 9). Correspondingly, the ROC method yields the highest ratio, while the AW method provides a value positioned in the intermediate range. Following the same analytical procedure as Example 1, Spearman rank correlation coefficients were calculated to compare the AW method with

the RR, RS, and ROC methods across five different MCDM ranking frameworks. These values are presented in Table IX.

According to Table IX, the Spearman correlation coefficients between the AW method and the RR, RS, and ROC methods are consistently equal to 1 across all five MCDM techniques. This indicates that the rankings obtained using the AW method are identical to those produced by the

individual component methods. In this case, the AW method fully achieves consensus among decision-makers, despite their differing preferences for RR, RS, or ROC. Across the three examined cases, including two synthetic examples and one real-world application, variations were observed in the number of alternatives as well as in the composition of benefit and cost criteria.

TABLE IX. SPEARMAN CORRELATION COEFFICIENTS BETWEEN AW AND OTHER WEIGHTING METHODS IN EXAMPLE 3

MCDM method	Weight method			Average
	RS	RR	ROC	
FUCA	1	1	1	1
SAW	1	1	1	1
PIV	1	1	1	1
ROV	1	1	1	1
RAM	1	1	1	1
Average				1

Nevertheless, all results demonstrate that the AW method produces rankings highly compatible with those obtained using RR, RS, and ROC. A further finding is related to the dispersion of criteria weights. The ROC method yields the highest ratio between maximum and minimum weights, indicating greater dispersion, while the RR and RS methods produce a fixed ratio equal to the number of criteria. In contrast, the AW method generates intermediate values, reflecting a balanced distribution of weights. This confirms that AW effectively integrates the characteristics of its component methods. Overall, the AW method can be regarded as a compromise weighting approach that mitigates the extremes of its individual components. By doing so, it reconciles differing methodological preferences and facilitates consensus in criteria weighting. To further assess the

TABLE XI. SCENARIOS IN EXAMPLE 2

Scenario	Priority order												Spearman
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	
S1	1	2	3	4	5	6	7	8	9	10	11	12	0.9919
S2	2	1	3	4	5	6	7	8	9	10	11	12	0.9677
S3	2	3	1	4	5	6	7	8	9	10	11	12	0.9927
S4	2	3	4	1	5	6	7	8	9	10	11	12	0.8174
S5	2	3	4	5	1	6	7	8	9	10	11	12	0.9927
S6	2	3	4	5	6	1	7	8	9	10	11	12	0.9895
S7	2	3	4	5	6	7	1	8	9	10	11	12	0.9976
C8	2	3	4	5	6	7	8	1	9	10	11	12	0.9095
C9	2	3	4	5	6	7	8	9	1	10	11	12	0.6606
S10	2	3	4	5	6	7	8	9	10	1	11	12	0.9927
S11	2	3	4	5	6	7	8	9	10	11	1	12	0.9814
S12	2	3	4	5	6	7	8	9	10	11	12	1	0.7657

In Example 2 (Table XI), most average Spearman coefficients are high. Exceptions occur in scenarios S4, S9, and S12, where the average values are relatively lower. Nonetheless, in most cases, 9 out of 12 scenarios, the average Spearman coefficients exceed 0.9. Since these values are well above the 0.8 threshold [29], it can be concluded that the rankings of alternatives are highly consistent when criteria weights are determined using the AW method compared to the other three methods.

For Example 3 (Table XII), the average Spearman coefficients in the first four scenarios are equal to 1, indicating

robustness of the proposed method, a comprehensive sensitivity analysis is conducted in the following section.

IV. SENSITIVITY ANALYSIS

To evaluate the performance of the AW method, a series of scenarios was generated for each example, corresponding to the total number of criteria involved. For Example 1, six scenarios were established by systematically rotating the primary criterion (sequentially assigning each criterion from C1 to C6 the highest priority) while maintaining the relative order of the remaining criteria, as summarized in Table X. Similarly, twelve scenarios were developed for Example 2 (Table XI), and nine for Example 3 (Table XII). In Tables XI and XII, Scenario S1 refers to the initial case analyzed above. The final column presents the mean Spearman correlation coefficients across the five MCDM methods and all four weighting techniques.

TABLE X. SCENARIOS IN EXAMPLE 1

Scenario	Priority order						Spearman
	C1	C2	C3	C4	C5	C6	
S1	1	2	3	4	5	6	0.9915
S2	2	1	3	4	5	6	0.9792
S3	2	3	1	4	5	6	0.9812
S4	2	3	4	1	5	6	0.9909
S5	2	3	4	5	1	6	0.993
S6	2	3	4	5	6	1	0.9525

In Example 1 (Table X), all average Spearman coefficients are high, with the lowest value being 0.9525 in scenario S6—well above the 0.8 threshold [29]. This demonstrates that using the AW method for criteria weighting produces results in alternative rankings that closely align with those obtained using the other three methods.

perfect agreement. In the remaining five scenarios, the coefficients remain high, with the lowest value being 0.9067 in scenario S7, which still significantly exceeds the 0.8 threshold [29]. These results confirm that alternative rankings, derived using the AW method, are highly consistent with those obtained from the RR, RS, and ROC methods. Overall, across the three examples and their corresponding sensitivity analyses, the findings demonstrate that the AW method produces rankings equivalent to or highly like those generated by its component methods. This is particularly important in situations where decision-makers differ in their choice of weighting

technique. In such cases, the AW method ensures reliable ranking outcomes while accommodating individual methodological preferences. Therefore, the AW method can be regarded as a compromise weighting approach that effectively integrates RR, RS, and ROC, promoting both accuracy and consensus in criteria weighting.

TABLE XII. SCENARIOS IN EXAMPLE 3

Scenario	Priority order									Spearman
	C1	C2	C3	C4	C5	C6	C7	C8	C9	
S1	1	2	3	4	5	6	7	8	9	1
S2	2	1	3	4	5	6	7	8	9	1
S3	2	3	1	4	5	6	7	8	9	1
S4	2	3	4	1	5	6	7	8	9	1
S5	2	3	4	5	1	6	7	8	9	0.9467
S6	2	3	4	5	6	1	7	8	9	0.9467
S7	2	3	4	5	6	7	1	8	9	0.9067
S8	2	3	4	5	6	7	8	1	9	0.9467
S9	2	3	4	5	6	7	8	9	1	0.96

## V. CONCLUSIONS

The Aggregated Weighting (AW) method represents an integrated framework that synthesizes three established subjective weighting techniques: Rank Reciprocal (RR), Rank Sum (RS), and Rank Order Centroid (ROC). By employing the geometric mean for aggregation, the AW approach eliminates the need for subjective priority coefficients, thereby effectively balancing diverse expert perspectives. This method promotes a robust consensus among decision-makers, particularly in contexts where there is divergence regarding the preferred ordinal weighting formula. Conceptually, AW functions as a compromise solution that moderates the inherent characteristics of its component methods. While ROC often results in high weight dispersion (exhibiting the highest max/min weight ratios), RR and RS are constrained by a rigid ratio equivalent to the number of criteria. In contrast, AW yields a balanced distribution, consistently maintaining a ratio that falls between these two extremes. A primary limitation of the current study is the underlying assumption that decision-makers have already reached a consensus on the rank ordering of criteria. In practical applications, such an agreement may not always be present. Consequently, extending the AW framework to accommodate scenarios involving conflicting criterion rankings represents a significant avenue for future research to enhance its applicability in complex real-world decision-making.

### DECLARATION OF COMPETING INTERESTS

Not applicable to this work.

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### DATA AVAILABILITY

Utilized data can be made available from the corresponding author upon request.

## AI USE AND DECLARATION OF GENERATIVE AI USE

The authors confirm that they did not use artificial intelligence technologies in creating the submitted work.

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