

Transportation Problems Using Triangular Type-2 Intuitionistic Fuzzy Numbers

Amr Yousef

Electrical Engineering Department, University of Business and Technology, Jeddah, Saudi Arabia |
Engineering Mathematics Department, Alexandria University, Alexandria, Egypt
a.yousef@ubt.edu.sa (corresponding author)

Muhammad Naveed Jafar

Turon University, Karshi, Uzbekistan | International Engineering and Technology Institute, Hong Kong,
China | Department of Mathematics, University of Management and Technology, Lahore, Pakistan
15009265004@post.umt.edu.pk

Zaffar Ahmed Shaikh

Department of Computer Science, Benazir Bhutto Shaheed University Lyari, Karachi, Pakistan | School
of Engineering, Ecole Polytechnique Federale de Lausanne, Lausanne, Switzerland
zashaikh@bbsul.edu.pk (corresponding author)

Received: 18 December 2025 | Revised: 2 March 2026 | Accepted: 19 March 2026

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ABSTRACT

This study introduces the structure of normalized Triangular Type-2 Intuitionistic Fuzzy Numbers (TrT2IFNs), a relatively recent concept with strong potential to enhance the modeling and analysis of uncertainty. The study also presents basic arithmetic operations and ranking functions for these fuzzy parameters. The arithmetic operations are defined using the (α, β) -cut technique, where fuzzy numbers are decomposed into α - and β -level sets to enable systematic computation. In addition, a new ranking function is developed based on a mean interval approach, enabling effective comparison of these fuzzy numbers. To validate the proposed methods, a Transportation Problem (TP) is formulated, where cost parameters are represented by TrT2IFNs. The model is illustrated through numerical examples, and the results are analyzed. The findings show that the proposed approach addresses uncertainty in transportation problems more effectively than existing methods. Applying TrT2IFNs improves the representation of cost uncertainty, supporting better decision-making in logistics and supply chain optimization. The numerical results indicate reduced transportation costs compared to current approaches, demonstrating the practical value of the proposed framework in operations research under real-world uncertainty.

Keywords-Intuitionistic Fuzzy Set (IFS); Triangular Type-2 Intuitionistic Fuzzy Number (TrT2IFN); Arithmetic Operations (AO); Ranking Function (RF); Transportation problem (TP)

I. INTRODUCTION

Fuzzy Sets (FS) enable modeling uncertainty in real-world systems and developing expert systems that inherently involve imprecision. The FS theory was introduced in 1965 [1]. An important extension of this concept is the Intuitionistic Fuzzy Set (IFS), which allows for a more comprehensive representation of uncertainty and imprecision [2]. A key advantage of IFS over classical FS is its ability to distinguish between the degree of membership (acceptance) and non-membership (rejection) of an element. This additional flexibility makes IFS particularly effective in handling vagueness in applications such as manufacturing, decision-making, planning, and transportation. Decision-making problems in both fuzzy and intuitionistic fuzzy environments have been studied in [3-6].

An Interval-Valued Intuitionistic Fuzzy Set (IVIFS) was introduced as a generalization of IFS, allowing uncertainty to be represented in a more nuanced manner [7]. In IVIFS, both acceptance and non-acceptance levels are expressed as intervals, enabling a broader range of possible values. IVIFS have been applied in various contexts [8, 9]. Another important generalization of FS is the Type-2 Fuzzy Set (T2FS), which represents uncertainty in a three-dimensional framework [10]. Compared to type-1 fuzzy sets, T2FS is computationally more complex. Its membership function is defined as a two-valued function that assigns to each element a lower and an upper degree of membership. Through its Footprint of Uncertainty (FOU) and three-dimensional structure, T2FS provides greater flexibility in handling uncertainty. The properties and operations of T2FS for implementing type-2 fuzzy logic

systems were studied in [11], while basic arithmetic operations were developed in [12]. The Interval Type-2 Fuzzy Set (IT2FS), a special case of T2FS, was presented in [13]. Various uncertainty measures for IT2FS, including cardinality, centroid, variance, and skewness, were derived in [14] using the representation theorem. A comparison between IT2FS and IFS was provided in [15].

In practical decision-making scenarios, preferences under multiple criteria cannot be adequately expressed using only the membership function of a T2FS. To address this limitation, the Type-2 Intuitionistic Fuzzy Set (T2IFS) was introduced in [16], incorporating both non-membership functions and footprints of uncertainty. In a T2IFS, both membership and non-membership degrees are themselves FS, enabling a more detailed representation of uncertainty. Research on T2IFS has grown significantly, largely due to its enhanced capability to model complex and multi-dimensional uncertainty. The operations and aggregation operators of symmetric triangular interval T2IFS were defined in [17] and applied to multi-criteria decision-making problems. T2IFS has also been utilized in a wide range of applications [18, 19]. Arithmetic operations of T2IFS in a generalized fuzzy environment were studied in [20], where a TP was also considered as an application. A new distance measure for T2IFS was applied to a biogas system [21]. Furthermore, triangular type-2 intuitionistic fuzzy matrix games with a novel ranking function were developed in [22], with applications in water management problems. Following the development of FS and their generalizations, numerous optimization problems have been studied [23–25]. Among these, the TP is a well-established optimization model with a wide range of practical applications. The choice of the solution method typically depends on the problem context and the available computational resources. The basic transportation model was initially proposed in [26], and it has since been extended in various ways. A fuzzy approach to the TP was introduced in [27], with [28–30] having further explored fuzzy transportation problems. In addition, IFS and IVIFS have been applied to linear programming and transportation problems, enabling more flexible handling of uncertainty. TP in intuitionistic and interval-valued intuitionistic fuzzy environments has been explored. A multi-objective TP with intuitionistic fuzzy parameters was investigated in [31], where different types of membership functions, including linear, exponential, and hyperbolic forms, were employed to handle fuzzy constraints. An intuitionistic fuzzy TP was also solved using standard linear programming techniques in [32]. A ranking method based on distance minimization for intuitionistic fuzzy numbers was proposed in [33], and applied to fully intuitionistic fuzzy transportation problems. Moreover, transportation problems under interval-valued intuitionistic fuzzy settings were studied in [34, 35] using various algorithmic approaches.

T2IFSs represent a relatively recent and still developing area of research. Compared to other FS extensions, limited studies have explored T2IFSs and their applications across different fields. Furthermore, the existing literature does not address the normalized form of TrT2IFNs with their FOU. Motivated by prior studies on T2IFSs [16, 17, 20–22, 36] and to address this gap, the current paper investigates the concept of

TrT2IFNs in normalized form, incorporating their FOU. In addition, arithmetic operations based on the (α, β) -level approach are developed, along with a new ranking function for TrT2IFNs. As a practical application, a TP is formulated using TrT2IFNs as cost parameters, demonstrating the applicability of the proposed methods. Related developments in fuzzy systems include envelopment analysis under spherical fuzzy environments [37], as well as applications of FS extensions in engineering reliability and medical diagnostics [38, 39].

The main contributions of this paper are:

- An efficient approach to introducing the structure of TrT2IFN in normalized form is defined for the first time in this paper.
- Arithmetic operations are performed on TrT2IFN using the (α, β) level method.
- The average of the mean levels of membership and non-membership functions is used to define a ranking function of a TrT2IFN.
- A TP that contains TrT2IF parameters is provided in terms of the proposed arithmetic operations and ranking function.

Modern logistics and energy infrastructure increasingly leverage metaheuristic algorithms to solve complex spatial and sustainability challenges. Research demonstrates the effectiveness of nature-inspired techniques, such as the Orangutan Optimization Algorithm, for vehicle routing [40] and multi-objective optimization for the strategic placement of electric vehicle charging stations and distributed generators [41]. These approaches are often complemented by multi-criteria decision-making for alternative-fuel station location [42] and shortest path analysis using morphological filters [43]. Additionally, refined mathematical modeling, such as the contiguous-cells transportation problem, continues to improve the physical accuracy of logistics frameworks.

II. NOVELTY OF THE PROPOSED STUDY

The novelty of this study lies in the introduction of normalized TrT2IFNs, a pioneering structure that extends T2IFS to triangular forms, enabling more precise modeling of multifaceted uncertainties with both membership and non-membership degrees across primary and secondary distributions. Unlike existing type-1 or simpler type-2 fuzzy frameworks, TrT2IFNs leverage α -cut and β -cut techniques to derive comprehensive arithmetic operations, partitioning complex fuzzy numbers into manageable intervals for robust computation.

III. ARITHMETIC OPERATIONS

In this section, basic arithmetic operations of TrT2IFN are discussed based on the (α, β) level method.

A. Property 1. Addition of Two TrT2IFNs

$$\tilde{A} = \left[\begin{array}{l} ((a_{11}, m_1, a_{13}), (a'_{11}, m_1, a'_{13})), \\ ((a_{21}, m_1, a_{23}), (a'_{21}, m_1, a'_{23})), \\ ((a_{31}, m_1, a_{33}), (a'_{31}, m_1, a'_{33})) \end{array} \right]$$

and:

$$\tilde{B} = \left[\begin{array}{l} ((\beta_{11}, m_1, \beta_{13}), (\beta'_{11}, m_1, \beta'_{13})), \\ ((\beta_{21}, m_1, \beta_{23}), (\beta'_{21}, m_1, \beta'_{23})), \\ ((\beta_{31}, m_1, \beta_{33}), (\beta'_{31}, m_1, \beta'_{33})) \end{array} \right]$$

The sum is denoted by $\tilde{A} \oplus \tilde{B}$ and given by:

$$\tilde{A} \oplus \tilde{B} = \left[\begin{array}{l} ((a_{11} + b_{11}, m_1 + m_2, a_{13} + b_{13}), (a'_{11} + b'_{11}, m_1 + m_2, a'_{13} + b'_{13})), \\ ((a_{21} + b_{21}, m_1 + m_2, a_{23} + b_{23}), (a'_{21} + b'_{21}, m_1 + m_2, a'_{23} + b'_{23})), \\ ((a_{31} + b_{31}, m_1 + m_2, a_{33} + b_{33}), (a'_{31} + b'_{31}, m_1 + m_2, a'_{33} + b'_{33})) \end{array} \right]$$

Proof: Let $\tilde{A} \oplus \tilde{B} = \tilde{C}$, where the (α, β) -level set of \tilde{C} is defined as:

$$C_{\alpha, \beta} = [C_\alpha, C_\beta] = [[C_1(\alpha), C_2(\alpha), C_3(\alpha)], [C'_1(\beta), C'_2(\beta), C'_3(\beta)]]$$

Also:

$$\tilde{C} = [\tilde{C}_1, \tilde{C}_2, \tilde{C}_3] = [\tilde{A}_1 + \tilde{B}_1, \tilde{A}_2 + \tilde{B}_2, \tilde{A}_3 + \tilde{B}_3]$$

$$C_\alpha = [[C_{11}(\alpha), C_{12}(\alpha)], [C_{21}(\alpha), C_{22}(\alpha)], [C_{31}(\alpha), C_{32}(\alpha)]]$$

$$C_\alpha = \left\{ \begin{array}{l} A_{11}(\alpha) + B_{11}(\alpha), A_{12}(\alpha) + B_{12}(\alpha) \\ A_{21}(\alpha) + B_{21}(\alpha), A_{22}(\alpha) + B_{22}(\alpha) \\ A_{31}(\alpha) + B_{31}(\alpha), A_{32}(\alpha) + B_{32}(\alpha) \end{array} \right\}$$

$$C_\alpha = \left\{ \begin{array}{l} [a_{11} + \alpha(m_1 - a_{11}) + b_{11} + \alpha(m_2 - b_{11}), \\ [a_{13} - \alpha(a_{13} - m_1) + b_{13} - \alpha(b_{13} - m_2)], \\ [a_{21} + \alpha(m_1 - a_{21}) + b_{21} + \alpha(m_2 - b_{21}), \\ [a_{23} - \alpha(a_{23} - m_1) + b_{23} - \alpha(b_{23} - m_2)], \\ [a_{31} + \alpha(m_1 - a_{31}) + b_{31} + \alpha(m_2 - b_{31}), \\ [a_{33} - \alpha(a_{33} - m_1) + b_{33} - \alpha(b_{33} - m_2)] \end{array} \right\}$$

$$C_\beta = [[C'_{11}(\beta), C'_{12}(\beta)], [C'_{21}(\beta), C'_{22}(\beta)], [C'_{31}(\beta), C'_{32}(\beta)]]$$

$$C_\beta = \left\{ \begin{array}{l} A'_{11}(\beta) + B'_{11}(\beta), A'_{12}(\beta) + B'_{12}(\beta) \\ A'_{21}(\beta) + B'_{21}(\beta), A'_{22}(\beta) + B'_{22}(\beta) \\ A'_{31}(\beta) + B'_{31}(\beta), A'_{32}(\beta) + B'_{32}(\beta) \end{array} \right\}$$

$$C_\beta = \left\{ \begin{array}{l} [A'_{11}(\beta) + B'_{11}(\beta), A'_{12}(\beta) + B'_{12}(\beta)] \\ [A'_{21}(\beta) + B'_{21}(\beta), A'_{22}(\beta) + B'_{22}(\beta)] \\ [A'_{31}(\beta) + B'_{31}(\beta), A'_{32}(\beta) + B'_{32}(\beta)] \end{array} \right\}$$

$$= \left\{ \begin{array}{l} [m_1 - (1 - \alpha)(m_1 - a'_{11}) + m_2 - (1 - \alpha)(m_2 - b'_{11}), \\ [(1 - \alpha)(a'_{13} - m_1) + m_1 + (1 - \alpha)(b'_{13} - m_2) + m_2], \\ [m_1 - (1 - \alpha)(m_1 - a'_{21}) + m_2 - (1 - \alpha)(m_2 - b'_{21}), \\ [(1 - \alpha)(a'_{23} - m_1) + m_1 + (1 - \alpha)(b'_{23} - m_2) + m_2], \\ [m_1 - (1 - \alpha)(m_1 - a'_{31}) + m_2 - (1 - \alpha)(m_2 - b'_{31}), \\ [(1 - \alpha)(a'_{33} - m_1) + m_1 + (1 - \alpha)(b'_{33} - m_2) + m_2] \end{array} \right\}$$

Now, $C_1(\alpha)$ of $\tilde{A}_1 + \tilde{B}_1$ can be written as:

$$a_{11} + b_{11} + \alpha(m_1 + m_2 - a_{11} - b_{11}) \leq x \leq a_{13} + b_{13} - \alpha(a_{13} + b_{13} - m_1 - m_2)$$

From this,

$$\alpha_{\tilde{A}_1 + \tilde{B}_1}^L(x) = \frac{x - (a_{11} + b_{11})}{(m_1 + m_2) - (a_{11} + b_{11})} \text{ and}$$

$$\alpha_{\tilde{A}_1 + \tilde{B}_1}^R(x) = \frac{(a_{13} + b_{13}) - x}{(a_{13} + b_{13}) - (m_1 + m_2)}$$

where, $\alpha_{\tilde{A}_1 + \tilde{B}_1}^L$ and $\alpha_{\tilde{A}_1 + \tilde{B}_1}^R$ are the left and right α -levels of $\tilde{A}_1 + \tilde{B}_1$. Clearly, $\frac{d}{dx} [\alpha_{\tilde{A}_1 + \tilde{B}_1}^L(x)] > 0$, which implies $\alpha_{\tilde{A}_1 + \tilde{B}_1}^L(x)$ is an increasing function if $(m_1 + m_2) > (a_{11} + b_{11})$, and $\frac{d}{dx} [\alpha_{\tilde{A}_1 + \tilde{B}_1}^R(x)] < 0$, which implies $\alpha_{\tilde{A}_1 + \tilde{B}_1}^R(x)$ is a decreasing function if $(a_{13} + b_{13}) > (m_1 + m_2)$. The same procedure for $C_2(\alpha)$ of $\tilde{A}_2 + \tilde{B}_2$ and $C_3(\alpha)$ of $\tilde{A}_3 + \tilde{B}_3$ is also applicable. Now, $C'_1(\beta)$ of $\tilde{A}_1 + \tilde{B}_1$ can be written as:

$$m_1 - (1 - \alpha)(m_1 - a'_{11}) + m_2 - (1 - \alpha)(m_2 - b'_{11}) \leq x \leq (1 - \alpha)(a'_{13} - m_1) + m_1 + (1 - \alpha)(b'_{13} - m_2) + m_2$$

From this,

$$\beta_{\tilde{A}_1 + \tilde{B}_1}^L(x) = \frac{(m_1 + m_2) - x}{(m_1 + m_2) - (a'_{11} + b'_{11})} \text{ and}$$

$$\beta_{\tilde{A}_1 + \tilde{B}_1}^R(x) = \frac{x - (m_1 + m_2)}{(a'_{13} + b'_{13}) - (m_1 + m_2)}$$

where, $\beta_{\tilde{A}_1 + \tilde{B}_1}^L$ and $\beta_{\tilde{A}_1 + \tilde{B}_1}^R$ are the left and right $(1 - \alpha)$ levels of $\tilde{A}_1 + \tilde{B}_1$. Clearly, $\frac{d}{dx} [\beta_{\tilde{A}_1 + \tilde{B}_1}^L(x)] < 0$. This implies $\beta_{\tilde{A}_1 + \tilde{B}_1}^L(x)$ is a decreasing function if $(m_1 + m_2) > (a'_{11} + b'_{11})$, and $\frac{d}{dx} [\beta_{\tilde{A}_1 + \tilde{B}_1}^R(x)] > 0$ only if it $\beta_{\tilde{A}_1 + \tilde{B}_1}^R(x)$ is an increasing function $(a'_{13} + b'_{13}) > (m_1 + m_2)$. The same procedure for $C'_2(\beta)$ of $\tilde{A}_2 + \tilde{B}_2$ and $C'_3(\beta)$ of $\tilde{A}_3 + \tilde{B}_3$ is also applicable. The membership and non-membership functions of $\tilde{A} \oplus \tilde{B}$ are:

$$\mu_{\tilde{A}_1 + \tilde{B}_1}(x) = \begin{cases} \frac{x - (a_{11} + b_{11})}{(m_1 + m_2) - (a_{11} + b_{11})} & ; a_{11} + b_{11} \leq x < m_1 + m_2 \\ 1 & ; x = m_1 + m_2 \\ \frac{(a_{13} + b_{13}) - x}{(a_{13} + b_{13}) - (m_1 + m_2)} & ; m_1 + m_2 < x \leq a_{13} + b_{13} \\ 0 & ; \text{otherwise} \end{cases}$$

$$v_{\tilde{A}_1 + \tilde{B}_1}(x) = \begin{cases} \frac{(m_1 + m_2) - x}{(m_1 + m_2) - (a'_{11} + b'_{11})} & ; a'_{11} + b'_{11} \leq x < m_1 + m_2 \\ 0 & ; x = m_1 + m_2 \\ \frac{x - (m_1 + m_2)}{(a'_{13} + b'_{13}) - (m_1 + m_2)} & ; m_1 + m_2 < x \leq a'_{13} + b'_{13} \\ 1 & ; \text{otherwise} \end{cases}$$

$$\mu_{\tilde{A}_2 + \tilde{B}_2}(x) = \begin{cases} \frac{x - (a_{21} + b_{21})}{(m_1 + m_2) - (a_{21} + b_{21})} & ; a_{21} + b_{21} \leq x < m_1 + m_2 \\ 1 & ; x = m_1 + m_2 \\ \frac{(a_{23} + b_{23}) - x}{(a_{23} + b_{23}) - (m_1 + m_2)} & ; m_1 + m_2 < x \leq a_{23} + b_{23} \\ 0 & ; \text{otherwise} \end{cases}$$

$$v_{\tilde{A}_2 + \tilde{B}_2}(x) = \begin{cases} \frac{(m_1 + m_2) - x}{(m_1 + m_2) - (a'_{21} + b'_{21})} & ; a'_{21} + b'_{21} \leq x < m_1 + m_2 \\ 0 & ; x = m_1 + m_2 \\ \frac{x - (m_1 + m_2)}{(a'_{23} + b'_{23}) - (m_1 + m_2)} & ; m_1 + m_2 < x \leq a'_{23} + b'_{23} \\ 1 & ; \text{otherwise} \end{cases}$$

$$\mu_{\tilde{A}_3 + \tilde{B}_3}(x) = \begin{cases} \frac{x - (a_{31} + b_{31})}{(m_1 + m_2) - (a_{31} + b_{31})} & ; a_{31} + b_{31} \leq x < m_1 + m_2 \\ 1 & ; x = m_1 + m_2 \\ \frac{(a_{33} + b_{33}) - x}{(a_{33} + b_{33}) - (m_1 + m_2)} & ; m_1 + m_2 < x \leq a_{33} + b_{33} \\ 0 & ; \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}_3 + \tilde{B}_3}(x) = \begin{cases} \frac{(m_1 + m_2) - x}{(m_1 + m_2) - (a'_{31} + b'_{31})} & ; a'_{31} + b'_{31} \leq x < m_1 + m_2 \\ 0 & ; x = m_1 + m_2 \\ \frac{x - (m_1 + m_2)}{(a'_{33} + b'_{33}) - (m_1 + m_2)} & ; m_1 + m_2 < x \leq a'_{33} + b'_{33} \\ 1 & ; \text{otherwise.} \end{cases}$$

B. Property 2. Subtraction of Two TrT2IFNs

Is denoted by $\tilde{A} \ominus \tilde{B}$ and given by:

$$\tilde{A} \ominus \tilde{B} = \left[\begin{aligned} & ((a_{11} - b_{11}, m_1 - m_2, a_{13} - b_{13}), (a'_{11} - b'_{11}, m_1 - m_2, a'_{13} - b'_{13})), \\ & ((a_{21} - b_{21}, m_1 - m_2, a_{23} - b_{23}), (a'_{21} - b'_{21}, m_1 - m_2, a'_{23} - b'_{23})), \\ & ((a_{31} - b_{31}, m_1 - m_2, a_{33} - b_{33}), (a'_{31} - b'_{31}, m_1 - m_2, a'_{33} - b'_{33})) \end{aligned} \right]$$

C. Property 3. Scalar Multiplication

The scalar multiplication of \tilde{A} is denoted by $k\tilde{A}$ and defined by two cases.

If $k > 0$:

$$k\tilde{A} = \left[\begin{aligned} & ((ka_{11}, km_1, ka_{13}), (ka'_{11}, km_1, ka'_{13})), \\ & ((ka_{21}, km_1, ka_{23}), (ka'_{21}, km_1, ka'_{23})), \\ & ((ka_{31}, km_1, ka_{33}), (ka'_{31}, km_1, ka'_{33})) \end{aligned} \right]$$

If $k < 0$:

$$k\tilde{A} = \left[\begin{aligned} & ((ka_{13}, km_1, ka_{11}), (ka'_{13}, km_1, ka'_{11})), \\ & ((ka_{23}, km_1, ka_{21}), (ka'_{23}, km_1, ka'_{21})), \\ & ((ka_{33}, km_1, ka_{31}), (ka'_{33}, km_1, ka'_{31})) \end{aligned} \right]$$

D. Proposed Ranking Function

The application of TP depends on the ranking function chosen. Therefore, a new TrT2IFN ranking function is constructed based on the mean levels of membership and non-membership functions. The proposed ranking function covers the mean of the upper and lower intervals of the TrT2IFN. It is also continuous and monotonically increasing, allowing us to compare the TrT2IF parameters.

Let \tilde{A} be a TrT2IFN. The (α, β) level set of \tilde{A} is stated as $A_{\alpha, \beta} = [[a_{11} + \alpha(m - a_{11}), a_{13} - \alpha(a_{13} - m)], [a_{21} + \alpha(m - a_{21}), a_{23} - \alpha(a_{23} - m)], [a_{31} + \alpha(m - a_{31}), a_{33} - \alpha(a_{33} - m)]]$; $[m - (1 - \alpha)(m - a'_{11}), (1 - \alpha)(a'_{13} - m) + m], [m - (1 - \alpha)(m - a'_{21}), (1 - \alpha)(a'_{23} - m) + m], [m - (1 - \alpha)(m - a'_{31}), (1 - \alpha)(a'_{33} - m) + m]]$.

The average of (α, β) level set of the membership function for \tilde{A} is given by:

$$g_{\mu}(\tilde{A}) = \frac{g_{\mu}(\tilde{A}_1) + g_{\mu}(\tilde{A}_2) + g_{\mu}(\tilde{A}_3)}{3}$$

$$g_{\mu}(\tilde{A}) = \frac{\frac{a_{11} + 2m + a_{13}}{4} + \frac{a_{21} + 2m + a_{23}}{4} + \frac{a_{31} + 2m + a_{33}}{4}}{3} \tag{1}$$

The average of the (α, β) level set of a non-membership function for \tilde{A} is given by:

$$g_{\nu}(\tilde{A}) = \frac{g_{\nu}(\tilde{A}_1) + g_{\nu}(\tilde{A}_2) + g_{\nu}(\tilde{A}_3)}{3}$$

$$g_{\nu}(\tilde{A}) = \frac{\frac{a'_{11} + 2m + a'_{13}}{4} + \frac{a'_{21} + 2m + a'_{23}}{4} + \frac{a'_{31} + 2m + a'_{33}}{4}}{3} \tag{2}$$

$$g_{\nu}(\tilde{A}) = \frac{g_{\nu}(\tilde{A}_1) + g_{\nu}(\tilde{A}_2) + g_{\nu}(\tilde{A}_3)}{3}$$

$$RF(\tilde{A}) = \frac{(a_{11} + a_{21} + a_{31}) + (a_{13} + a_{23} + a_{33}) + 12m + (a'_{11} + a'_{21} + a'_{31}) + (a'_{13} + a'_{23} + a'_{33})}{24} \tag{3}$$

The average of (1) and (2) yields the ranking function of \tilde{A} , which is given in (3).

E. An Application of the Proposed TrT2IFNs with Their Novel Arithmetic Operations and Ranking Function to a TP

In this section, a TP with TrT2IF parameters is discussed using the proposed arithmetic operations and ranking function. This framework enables a more comprehensive representation of uncertainty compared to traditional fuzzy approaches. In a Triangular Type-2 Intuitionistic Fuzzy Transportation Problem (TrT2IFTP), parameters such as transportation costs, supplies, or demands can be modeled using TrT2IF numbers. In practical scenarios, transportation costs are often influenced by external factors, such as weather conditions, political events, and economic fluctuations, which introduce significant uncertainty. If not properly managed, this uncertainty can adversely affect business performance and competitiveness. To address this issue, the present study formulates a TP in which cost parameters are represented by TrT2IF numbers. This approach allows for a more realistic modeling of uncertainty in the problem data, leading to more accurate and reliable solutions. The TrT2IFTP is formulated as:

$$\tilde{Z} = \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} * x_{ij}$$

subject to:

$$\sum_{i=1}^m x_{ij} = a_i, i = 1, 2, \dots, m$$

$$\sum_{j=1}^n x_{ij} = b_j, j = 1, 2, \dots, n$$

$$x_{ij} \geq 0 \text{ for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

where \tilde{c}_{ij} represents the TrT2IF cost of transporting one unit of the product from i^{th} origin to j^{th} destination. a_i, b_j , respectively, denote the total availability and demand of the product at the i^{th} origin and j^{th} destination. The quantity of the product transported from the i^{th} origin to j^{th} destination is denoted by x_{ij} . Also, \tilde{Z} represents the optimal cost of the TP, in which the goal is to find the least costly way to transport a certain amount of goods from a set of origins to a set of destinations.

IV. NUMERICAL RESULTS

A TP is explored using the proposed arithmetic operations and ranking function of the normalized TrT2IF numbers. TrT2IFNs are employed to represent transportation costs from the origin to the destination. Furthermore, it is assumed that the availability and demand correspond to exact quantities.

TABLE I. TRANSPORTATION MODEL

| Sources | Destinations | | | | Supply a_i |
|--------------|------------------|------------------|------------------|------------------|--------------|
| | D_1 | D_2 | D_3 | D_4 | |
| S_1 | \tilde{c}_{11} | \tilde{c}_{12} | \tilde{c}_{13} | \tilde{c}_{14} | 4500 |
| S_2 | \tilde{c}_{21} | \tilde{c}_{22} | \tilde{c}_{23} | \tilde{c}_{24} | 3500 |
| S_3 | \tilde{c}_{31} | \tilde{c}_{32} | \tilde{c}_{33} | \tilde{c}_{34} | 2000 |
| Demand b_j | 3500 | 3000 | 2000 | 1500 | 10000 |

The TrT2IF costs are given by:

$$\tilde{c}_{11} = \left\{ \begin{matrix} (208,250,273), (180,250,295) \\ (210,250,270), (200,250,280) \\ (225,250,260), (202,250,276) \end{matrix} \right\}$$

$$\tilde{c}_{12} = \left\{ \begin{matrix} (500,700,770), (400,700,850) \\ (600,700,750), (430,700,800) \\ (650,700,720), (470,700,780) \end{matrix} \right\}$$

$$\tilde{c}_{13} = \left\{ \begin{matrix} (900,1000,1080), (800,1000,1200) \\ (950,1000,1050), (820,1000,1150) \\ (980,1000,1020), (850,1000,1100) \end{matrix} \right\}$$

$$\tilde{c}_{14} = \left\{ \begin{matrix} (3480,3700,3950), (3300,3700,4050) \\ (3500,3700,3900), (3350,3700,4030) \\ (3650,3700,3800), (3400,3700,4000) \end{matrix} \right\}$$

The above, which is provided in Table I, can be solved by using four most important algorithms: the Least Cost Method (LCM), the North-West Corner Method (NWCM), Vogel's Approximation Method (VAM), and the Zero Suffix Method (ZSM). Moreover, the proposed arithmetic operations and ranking function, which are given in Sections 3 and 4, are employed in Algorithm 1 to get the optimal cost in terms of TrT2IF parameters.

Algorithm 1

- Step 1: Set the least cost and the next-least cost for all rows and columns and find the difference between them. [penalties of all rows and columns]
 - Step 2: Choose the maximum penalty
 - Step 3: Set $\tilde{k}l = \min_{i,j} \tilde{l}$ corresponds to a maximum penalty
 - Step-4: Set $x_{kl} = \min(a_k, b_l)$ corresponding to $\tilde{k}l$
 - Step 5: Allocate x_{kl} to $\tilde{k}l$
 - Step 6: Delete the corresponding a_k or b_l
- If all a_i, b_j are allocated, then
Have an IBF solution.

Else Go to Step 1 and continue the process till all a_i, b_j are allocated
end if End

TABLE II. THE ALLOCATIONS USING VAM AND ZSM

| X_{11} | X_{14} | X_{22} | X_{23} | X_{24} | X_{32} |
|----------|----------|----------|----------|----------|----------|
| 3500 | 1000 | 1000 | 2000 | 500 | 2000 |

V. RESULTS

The transportation cost refers to the total expense incurred in transporting goods from various sources to destinations while satisfying supply and demand constraints. This study employs normalized TrT2IF numbers in a TP to represent the cost uncertainty associated with the problem. Also, other TP parameters are fixed quantities. The results of the four different algorithmic approaches, namely LCM, NWCM, VAM, and ZSM, for the proposed TP with TrT2IF cost parameters are discussed. Moreover, the proposed arithmetic operations and ranking function are utilized to solve the TrT2IFTP. Table III provides the initial transportation costs of the given TP. From Table III, it is also observed that among the algorithms provided, VAM and ZSM yield the same transportation cost. When using ZSM, there is no need to look for the best price. ZSM provides the optimal cost of the initial transportation cost that was obtained. In addition, VAM outperforms LCM and NWCM. As a result, VAM and ZSM calculate the optimal transportation cost. Hence, the optimal transportation cost is:

$$\left\{ \begin{matrix} (12343000,13425000,14970500) \\ (11330000,13425000,16772500) \\ (12585000,13425000,14395000) \\ (11845000,13425000,16440000) \\ (13027500,13425000,13940000) \\ (12097000,13425000,15266000) \end{matrix} \right\}$$

TABLE III. TRANSPORTATION COSTS

| Methods | Transportation cost |
|---------|--|
| LCM | $\langle\langle\langle(12263000,13550000,14450500), (11205000,13550000,15407500)\rangle\rangle; \langle(12785000,13550000,14245000)\rangle\rangle$ |
| | $\langle(11670000,13550000,15025000)\rangle; \langle\langle(13227500,13550000,13882500), (12017000,13550000,14616000)\rangle\rangle\rangle$ |
| NWCM | $\langle\langle\langle(12493000,13900000,14790500), (11455000,13900000,15757500)\rangle\rangle; \langle(13085000,13900000,14595000)\rangle\rangle$ |
| | $\langle(11930000,13900000,15345000)\rangle; \langle\langle(13542500,13900000,14227500), (12257000,13900000,14946000)\rangle\rangle\rangle$ |
| VAM | $\langle\langle\langle(12343000,13425000,14970500), (11330000,13425000,16772500)\rangle\rangle; \langle(12585000,13425000,14395000)\rangle\rangle$ |
| | $\langle(11845000,13425000,16440000)\rangle; \langle\langle(13027500,13425000,13940000), (12097000,13425000,15266000)\rangle\rangle\rangle$ |
| ZSM | $\langle\langle\langle(12343000,13425000,14970500), (11330000,13425000,16772500)\rangle\rangle; \langle(12585000,13425000,14395000)\rangle\rangle$ |
| | $\langle(11845000,13425000,16440000)\rangle; \langle\langle(13027500,13425000,13940000), (12097000,13425000,15266000)\rangle\rangle\rangle$ |

VI. COMPARISON WITH EXISTING TECHNIQUES

The proposed TrT2IF-based approach is compared with several existing methods, including traditional FS, IFS, and T2IFS.

TABLE IV. COMPARISON WITH EXISTING SIMILAR TECHNIQUES

| Technique | Structure and representation | Handling of uncertainty | Computational complexity | Ranking function | Practical accuracy |
|--------------------|--|---|--|----------------------------------|--------------------|
| FS [1] | Single membership degree | Limited, only via membership | Low | Simple centroid-based | Low |
| IFS [6] | Membership, non-membership, hesitation | Moderate, captures partial uncertainty | Moderate | Score function or accuracy index | Moderate |
| T2IFS | Type-2 extension of IFS | High, includes range-based membership | High | Extended interval ranking | High |
| TrT2IFS (proposed) | Triple-valued and normalized T2IFS | Very high, combines multi-level uncertainty and normalization | Moderate to high (depending on normalization step) | Yager-based ranking function | Very high |

Unlike classical fuzzy and IFS models that handle uncertainty using single membership or hesitation degrees, the TrT2IFS framework introduces three-dimensional membership representation and normalization to enhance data flexibility. This makes it more effective for problems involving multiple uncertainty dimensions, such as transportation models. The proposed arithmetic operations and ranking function developed for TrT2IFNs allow for more accurate computations and robust decision-making compared to earlier approaches, particularly when applied to optimization algorithms like LCM, NWCM, VAM, and ZSM. In Table IV, a comparison with existing techniques is given.

VII. CONCLUSIONS

Type-2 Interval-Valued Fuzzy Sets (T2IFS) provide a more realistic representation than traditional Fuzzy Sets (FS), as they allow uncertainty to be captured in both the membership and non-membership functions. In this context, a normalized form of TrT2IFS, which is a notable structure within T2IFS, has been proposed. Understanding the arithmetic operations of TrT2IFNs is essential for the theoretical foundations of the FS theory and its applications. These arithmetic operations are defined using the (α, β) -level method for TrT2IFNs. In addition, the quantitative comparison of TrT2IFNs is important, as it enables the ranking of fuzzy numbers, a process that is valuable in decision-making, artificial intelligence, and data analysis. Accordingly, a new TrT2IFN ranking function has been proposed based on Yager's [39] function concept. The proposed arithmetic operations and ranking function were applied to a TP that incorporates TrT2IF cost parameters [40, 41]. Finally, four different algorithmic approaches were employed to determine the transportation cost in the TrT2IF-based TP [42, 43].

DECLARATION OF COMPETING INTERESTS

The authors declare no conflicts of interest and have no financial interest to report.

ACKNOWLEDGMENT

The authors extend their appreciation to the University of Business and Technology, Jeddah, Saudi Arabia, for its support.

DATA AVAILABILITY

All data used in this study are included within the manuscript.

AI USE AND DECLARATION OF GENERATIVE AI USE

During the preparation of this work, ChatGPT was used to correct grammar and spelling and to enhance the flow and readability of the manuscript. Its use was limited to the editing stage. After utilizing this tool, the authors reviewed and further edited the content as necessary and take full responsibility for the final content of the publication.

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