

A Multi-Objective Optimization and Sensitivity-Driven Decision Framework for Split-Output Two-Stage Helical Gearboxes Using NSGA-II and MCDM Methods

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ABSTRACT

This study presents an optimization and decision-support framework for the design of split-output two-stage helical gearboxes, integrating multi-objective evolutionary optimization with Multi-Criteria Decision-Making (MCDM) techniques. The proposed framework employs the Non-dominated Sorting Genetic Algorithm II (NSGA-II) to simultaneously minimize the gearbox length (L_{gb}) and maximize the gearbox efficiency (η_{gb}), generating a Pareto-optimal set of design alternatives. Three MCDM methods, MAIRCA, MARCOS, and Evaluation based on Average Ranking (EAMR) are utilized to rank the Pareto solutions based on varying weight distributions between the cost and benefit criteria. A sensitivity-driven analysis is conducted by systematically perturbing the weighting factors to evaluate the stability and robustness of each decision method under different trade-off conditions. The results reveal that while all three methods produce consistent trends, their sensitivity levels differ significantly. EAMR demonstrates the highest robustness, maintaining stable rankings across both local (per-transmission ratio u_h) and global analyses, whereas MARCOS shows greater responsiveness to weight changes and MAIRCA demonstrates intermediate sensitivity. Heatmap-based visualization of global Top-1 frequencies and stability indices further confirms EAMR's superiority in delivering reliable, reproducible decisions. Overall, the proposed framework not only ensures an effective balance between compactness and efficiency in gearbox design but also provides a quantitative means to assess decision robustness across multiple MCDM strategies. The

proposed methodology can be extended to other mechanical systems requiring trade-off optimization between geometric, energetic, and performance objectives.

Keywords-two-stage helical gearbox; split output stage; multi-objective optimization; NSGA-II; MAIRCA; MARCOS; EAMR; sensitivity analysis; gearbox efficiency; gearbox length

I. INTRODUCTION

Gear transmission systems play a critical role in modern mechanical engineering, especially in applications requiring compactness, high efficiency, and durability. Helical gearboxes are widely used in industrial systems due to their smooth operation and superior load-carrying capacity compared with spur gearboxes [1, 2]. However, their design remains a challenging task as engineers must balance conflicting objectives such as minimizing volume or length while maximizing overall efficiency. Conventional design methods relying on empirical rules or single-objective formulations often fail to capture such trade-offs, motivating the adoption of advanced optimization frameworks [3, 4].

Numerous approaches have been introduced for gearbox optimization. Mathematical programming and heuristic search techniques have been employed, but their limited ability to handle nonlinearities and multiple objectives restricted their applicability [1]. With the emergence of evolutionary algorithms, significant progress has been achieved. NSGA-II, proposed in [5], has been widely recognized for its ability to generate well-distributed Pareto fronts. Various applications in engineering domains confirmed the robustness of NSGA-II in solving complex multi-objective optimization problems [6, 7]. In gearbox design, NSGA-II has been successfully applied to multi-speed systems [8] and spur gear optimization [9, 10]. Decision-making methods have been coupled with NSGA-II to identify the most preferred solution from Pareto fronts [10, 11].

In the specific context of helical gearboxes, optimization models have been developed considering volume, efficiency, vibration, and noise constraints. Authors in [12] combined the Taguchi method with grey relational analysis to optimize two-stage helical gearboxes, whereas authors in [13] applied MARCOS to evaluate trade-offs in systems with double gear sets in the first stage. Similarly, authors in [14] investigated a split first-stage configuration to minimize gearbox volume and improve efficiency, while authors in [1] focused on split first-stage gearboxes using the EAMR technique. Although these studies demonstrate the importance of considering split-stage layouts, they also reveal that most approaches have been limited either to specific gearbox architectures or to a single decision-making method.

Gearbox optimization objectives have been expanded to include vibration, acoustic, and tribological performance [4, 15, 16], confirming the flexibility of multi-objective formulations. However, the integration of systematic decision-making tools with evolutionary optimization remains limited in gearbox design. In particular, the comparative use and sensitivity assessment of modern MCDM methods, such as MAIRCA, MARCOS, and EAMR, have not yet been explored for helical gearbox optimization.

To fill this gap, the present study develops a multi-objective optimization and sensitivity-driven decision framework for

split-output two-stage helical gearboxes. The NSGA-II algorithm is employed to generate Pareto-optimal solutions that minimize gearbox length (L_{gb}) and maximize efficiency (η_{gb}), while the three MCDM methods are used to rank and select compromise/compromised designs under varying weighting scenarios. A sensitivity analysis is then performed to examine the stability and robustness of each decision method.

The main contributions of this study are threefold. First, a Pareto-based optimization model is developed for the design of split-output two-stage helical gearboxes to simultaneously minimize gearbox length and maximize efficiency. Second, the framework integrates the NSGA-II algorithm with multiple MCDM methods (MAIRCA, MARCOS, and EAMR) to enhance decision-making and identify optimal compromise solutions. Finally, a sensitivity-oriented evaluation is conducted to assess the stability and robustness of each decision method, revealing that EAMR consistently provides the most stable and reliable ranking for achieving compact and high-efficiency gearbox designs.

II. OPTIMIZATION PROBLEM

A. Determining Gearbox Length

For the two-stage helical gearbox with dual gears in the second stage, whose dimensions are shown in Figure 1, the length L can be calculated by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 4 \cdot \delta \quad (1)$$

where $\delta = 7 \div 10$ (mm), d_{w1i} , and d_{w2i} ($i = 1 \div 2$) denote the pitch diameter of the pinion and the gear of stage i , which were determined by [17]:

$$d_{w1i} = 2 \times \frac{a_{wi}}{(u_i+1)} \quad (2)$$

$$d_{w2i} = 2 \times \frac{a_{wi} \cdot u_i}{(u_i+1)} \quad (3)$$

where a_{wi} ($i = 1 \div 2$) is the center distance of stage i , which was computed by [17]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / ([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (4)$$

where X_{bai} is the wheel face width coefficient of stage i th and T_{1i} ($i = 1 \div 2$) is the pinion torque of stage i , which is calculated using:

$$T_{11} = \frac{T_r}{u_{gb} \cdot \eta_{hg}^2 \cdot \eta_{be}^3} \quad (5)$$

$$T_{12} = \frac{T_r}{2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2} \quad (6)$$

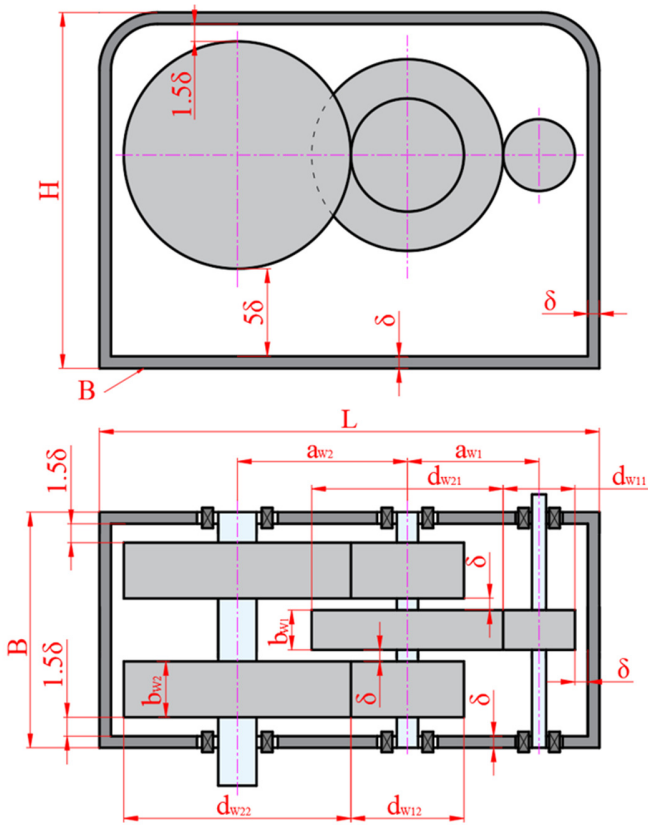


Fig. 1. Dimensions of the two-stage helical gearbox with dual gears.

B. Determining Gearbox Efficiency

The total gearbox efficiency η_{gb} is calculated using:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \tag{7}$$

where P_l is the overall power loss in the gearbox, which was determined by [18]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{z0} \tag{8}$$

where P_{lg} , P_{lb} , P_{ls} , and P_{z0} are the gear meshing loss, bearing friction loss, seal resistance loss, and the loss due to idle motion, respectively. These parameters can be determined as proposed in [12].

C. Objective Functions and Constraints

The design of the two-stage helical gearbox with a split output stage is formulated as a bi-objective optimization problem, where two conflicting goals must be addressed simultaneously: minimizing the overall gearbox length L to achieve a more compact structure, and maximizing the mechanical efficiency η_{gb} to ensure high transmission performance. The optimization problem is therefore expressed through the following objective functions:

$$\min f_1 = L \tag{9}$$

$$\max f_2 = \eta_{gb} \tag{10}$$

To achieve these objectives, the optimization is defined in terms of three continuous design variables that strongly influence both the geometric configuration and efficiency of the gearbox. These factors are u_1 , X_{ba1} , and X_{ba2} and they govern the distribution of gear ratios across the stages. They also determine the dimensional characteristics of the gear pairs, thereby directly affecting the housing length and power transmission efficiency.

To ensure the feasibility of the solutions and guarantee manufacturability, practical constraints are imposed on the design variables [17]:

$$1 \leq u_i \leq 9 \tag{11}$$

$$0.25 \leq X_{bai} \leq 0.4 \tag{12}$$

III. METHODOLOGY

A. NSGA II Method

To solve the formulated bi-objective optimization problem, the NSGA-II was employed due to its proven effectiveness in addressing multi-objective engineering design problems with conflicting objectives [7]. NSGA-II is particularly suitable for gearbox optimization, as it provides a diverse set of Pareto-optimal solutions that allow designers to explore trade-offs between compactness and efficiency.

The key features of NSGA-II that make it highly efficient include:

- Fast non-dominated sorting procedure, which classifies solutions into different Pareto fronts according to the concept of dominance.
- Crowding distance assignment, which preserves solution diversity by maintaining a well-distributed set of non-dominated points along the Pareto front.
- Elitism, which ensures that the best solutions from one generation are carried over to the next, preventing loss of high-quality solutions during the evolutionary process.

In the present study, the optimization process begins with an initial population of gearbox designs generated within the feasible ranges defined in (11) and (12). Each solution is evaluated based on the two objective functions, namely gearbox length L and efficiency η_{gb} . The NSGA-II algorithm then iteratively evolves the population through genetic operators:

- Selection: solutions are chosen based on Pareto rank and crowding distance.
- Crossover: selected parent solutions exchange design variable information to generate offspring.
- Mutation: small random perturbations are applied to design variables to maintain diversity and avoid premature convergence.

Over successive generations, the population converges toward the Pareto front, which represents the optimal trade-offs between minimizing gearbox length and maximizing efficiency. The final set of Pareto-optimal solutions provides

valuable decision support, allowing designers to select configurations that best satisfy specific requirements.

Three MCDM methods, MAIRCA, MARCOS, and EAMR are employed to rank the Pareto-optimal solutions generated by NSGA-II. These techniques provide systematic approaches for evaluating alternatives under conflicting criteria such as gearbox length (L_{gb}) and efficiency (η_{gb}). In general, each method involves normalizing the decision matrix, weighting the criteria, and calculating a composite score to determine the relative performance of each design alternative. MAIRCA evaluates alternatives based on the deviation from an expected performance level, MARCOS compares each alternative to both ideal and anti-ideal solutions, and EAMR applies linear normalization and weighted aggregation to achieve balanced rankings. The detailed computational procedures for these methods are provided in [19] for MAIRCA and EAMR, and in [20] for MARCOS, where their step-by-step formulations can be found.

IV. RESULTS AND DISCUSSION

The proposed multi-objective optimization and sensitivity-driven decision framework was applied to optimize the two-stage helical gearbox with a split output stage.

A. Mean Trends of Gearbox Length and Efficiency

The influence of the transmission ratio on the mean gearbox length and efficiency is illustrated in Figure 3.

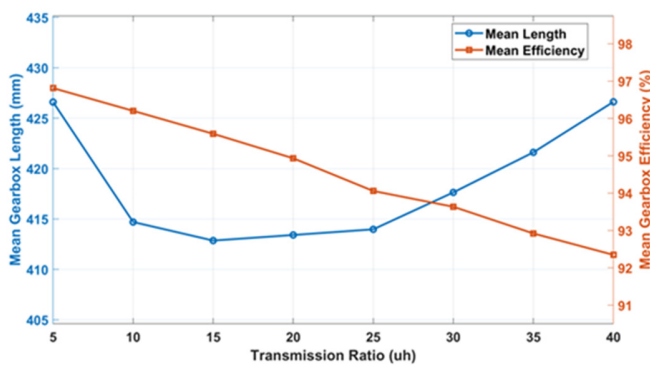


Fig. 2. Mean trends of gearbox length and efficiency with respect to u_h .

It is evident that increasing u_h reduces the mean efficiency η_{gb} , declining from approximately 97% at $u_h = 5$ to about 92% at $u_h = 40$. In contrast, the gearbox length L first decreases when u_h increases from 5 to 15, reaching its minimum around 408–410 mm, before gradually increasing again. This trade-off highlights a conflicting trend: higher transmission ratios improve compactness up to a certain point, but excessive ratios induce larger sizes and greater power losses.

B. Pareto-Optimal Fronts Across Transmission Ratios

Figure 4 displays the Pareto fronts obtained via NSGA-II for all examined values of u_h . Each front represents the non-dominated set of designs balancing gearbox length and efficiency. The curves show a typical concave trade-off: moving towards shorter gearbox lengths results in a reduction in efficiency, while maximizing efficiency requires

compromises in compactness. The Pareto fronts for intermediate transmission ratios ($u_h = 15$ to 25) exhibit better balance, providing feasible solutions with both relatively low gearbox length (around 405–415 mm) and high efficiency (>94%). In contrast, extreme ratios ($u_h = 5$ or $u_h = 40$) shift the fronts outward, reflecting less favorable trade-offs.

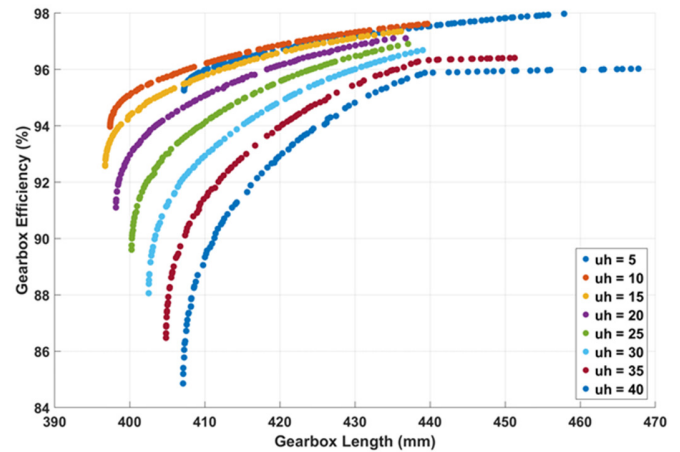


Fig. 3. Pareto fronts of gearbox length versus efficiency across transmission ratios.

C. Decision Ranking and Sensitivity Analysis of MCDM Methods

To evaluate the reliability of MCDM techniques in selecting the most appropriate gearbox configurations, three methods, MAIRCA, MARCOS, and EAMR, were compared through both ranking analysis and sensitivity evaluation. Each method was applied to the same Pareto set generated by NSGA-II, using the two design objectives: minimizing gearbox length (L_{gb}) and maximizing efficiency (η_{gb}).

1) Decision Ranking Behavior

After equal weighting of the two criteria ($w_L = w_\eta = 0.5$), all three methods yielded consistent rankings, locating the best compromise designs in the moderate ratio region ($u_h = 15$ to 25), where $L_{gb} \approx 400$ mm and $\eta_{gb} > 94\%$. However, when the weights were perturbed, differences appeared: MARCOS favored more compact (low- u_h) designs, EAMR preferred efficiency-oriented (high- u_h) solutions, while MAIRCA remained between these two extremes.

2) Sensitivity and Stability Evaluation

A sensitivity-driven analysis was performed to assess how each MCDM method reacts to changes in weighting factors.

In Figure 5, the stability heatmap indicates that EAMR maintains the smallest number of unique Top-1 alternatives across all weighting scenarios and transmission ratios, demonstrating its high robustness and repeatability. In contrast, MARCOS displays more variations (lighter color regions), confirming its higher responsiveness, whereas MAIRCA exhibits intermediate stability.

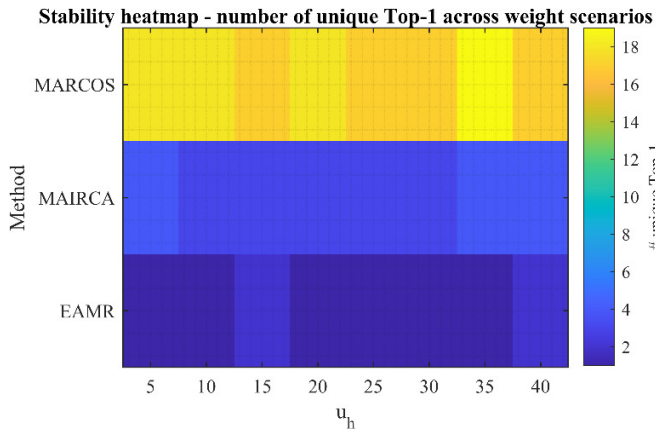


Fig. 4. Stability heatmap illustrating that EAMR achieves the highest-ranking stability across weighting scenarios.

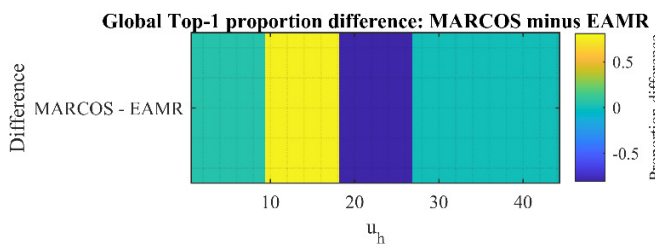


Fig. 5. Global Top-1 difference (MARCOS–EAMR) indicating MARCOS dominance at low u_h and EAMR dominance at high u_h .

3) Interpretation and Conclusion

These findings highlight a complementary relationship between MARCOS and EAMR: the former is more sensitive and useful for exploring design trade-offs, while the latter is more stable and dependable for the final decision-making. Considering its lowest variability, highest reproducibility, and strong correlation with Pareto-front trends, EAMR is ultimately proposed as the primary MCDM method for robust and consistent gearbox design selection.

D. Decision Support Using EAMR

While NSGA-II generates a wide range of Pareto-optimal solutions, a single compromise design must be identified for practical gearbox implementation. The EAMR method was employed to rank the Pareto solutions under the two conflicting criteria – gearbox length (L_{gb} , cost-type) and efficiency (η_{gb} , benefit-type). Table I lists the best compromise configurations selected by EAMR for each total transmission ratio (u_h), including the main design variables (u_1 , X_{ba1} , X_{ba2}) and corresponding performance indices.

The EAMR ranking confirms that moderate transmission ratios ($u_h = 10$ to 20) provide the most balanced designs, maintaining high efficiency (≈ 94.3 – 95.0%) with compact gearbox length (≈ 399 – 403 mm). At low ratios ($u_h=5$), efficiency reaches its maximum (95.71%), but the gearbox becomes slightly longer (≈ 408 mm), whereas at high ratios ($u_h \geq 35$), compactness improves marginally, but efficiency decreases to around 93.3% . Therefore, the range of $u_h = 10$ to 25 can be regarded as the most practical region for balancing compactness and energy efficiency.

TABLE I. BEST COMPROMISE SOLUTIONS SELECTED BY EAMR

u_h	u_1	X_{ba1}	X_{ba2}	L (mm)	η_{gb} (%)
5	1.93	0.40	0.40	408.16	95.71
10	2.73	0.40	0.40	399.20	94.95
15	3.29	0.40	0.40	399.81	94.33
20	3.72	0.40	0.40	402.68	93.84
25	4.14	0.40	0.40	406.31	93.37
30	4.41	0.38	0.40	411.54	93.31
35	4.88	0.36	0.40	416.70	93.30
40	5.08	0.34	0.40	421.60	93.31

Further examination of the geometric parameters reveals a distinct two-region behavior of X_{ba1} .

- For $u_h = 5$ to 25 , X_{ba1} remains constant at 0.40 , ensuring stable tooth-width proportions at moderate loads.
- For $u_h = 25$ to 40 , X_{ba1} decreases almost linearly, following the regression with $R^2 = 0.9888$, as depicted in Figure 5:

$$X_{ba1} = 0.5038 - 0.0041 \cdot u_h \tag{13}$$

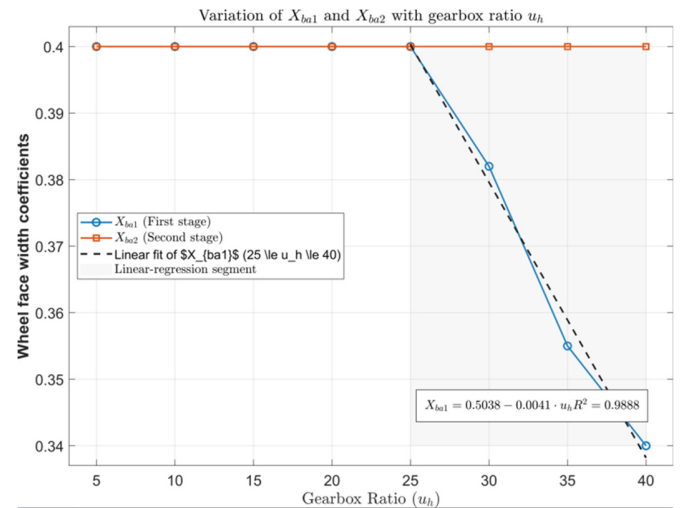


Fig. 6. Variation of X_{ba1} and X_{ba2} with a linear fit for $u_h \in (25, 40)$.

This behavior indicates that structural adjustment occurs mainly in the first stage, while the second-stage coefficient X_{ba2} remains constant at 0.40 , confirming its stabilizing role in load distribution.

In addition, a logarithmic correlation between the first-stage gear ratio u_1 and the total gear ratio u_h was observed for all EAMR-optimal designs, as portrayed in Figure 6:

$$u_1 = 1.5244 \cdot \ln(u_h) - 0.7034 \tag{14}$$

Equation (14) provides a simple predictive rule for determining the stage-1 gear ratio once the total ratio u_h has been chosen using EAMR. The high determination coefficient demonstrates that the logarithmic model fits the design trend accurately and can be used to guide future gearbox dimensioning.

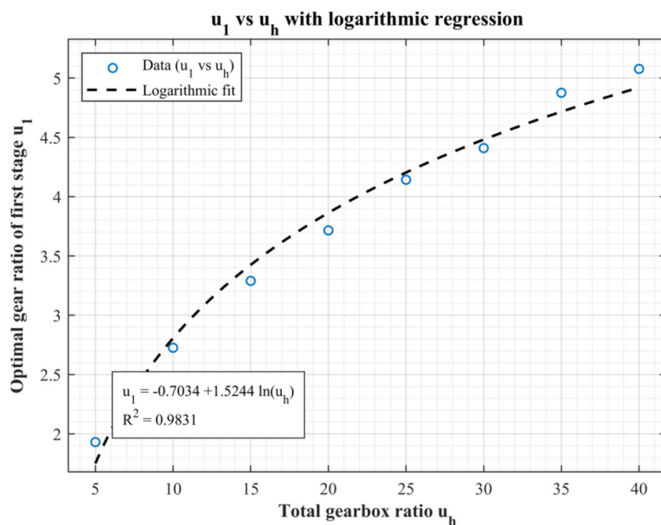


Fig. 7. Relationship between u_1 and u_h with logarithmic regression.

V. CONCLUSIONS

This study developed a multi-objective optimization and decision-support framework for the design of split-output two-stage helical gearboxes, integrating the Non-dominated Sorting Genetic Algorithm II (NSGA-II) with the Evaluation based on Average Ranking (EAMR) method. The NSGA-II algorithm was employed to generate Pareto-optimal solutions that minimize the gearbox length (L_{gb}) and maximize the efficiency (η_{gb}), while EAMR was applied to identify the most balanced configurations under varying transmission ratios. The main conclusions are:

- EAMR ranking revealed that moderate total transmission ratios of $u_h = 10$ to 20 yield the most balanced gearbox designs, maintaining high efficiency ($\approx 94\text{--}95\%$) with compact lengths ($\approx 399\text{--}403$ mm). At low ratios ($u_h = 5$), efficiency reaches its peak, but the gearbox becomes longer, whereas high ratios ($u_h \geq 35$) improve compactness but reduce efficiency to around 93%. Hence, $u_h = 10$ to 25 represents the most practical design range.
- The geometric parameters show that the second-stage face width coefficient remains constant at $X_{ba2} = 0.40$ for all ratios, while the first-stage coefficient X_{ba1} remains constant at 0.40 for $u_h = 10$ to 25 and decreases linearly for $u_h = 25$ to 40 following the relation described by (13). This indicates that dimensional adjustments mainly occur in the first stage, while the second stage remains structurally stable.
- A strong logarithmic correlation between the optimum gear ratio of the first-stage and total ratios was established by (14), providing a practical formula for determining the first-stage gear ratio once the total ratio is known.

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REFERENCES

- [1] H. Wang and H.-P. Wang, "Optimal Engineering Design of Spur Gear Sets," *Mechanism and Machine Theory*, vol. 29, no. 7, pp. 1071–1080, Oct. 1994, [https://doi.org/10.1016/0094-114X\(94\)90074-4](https://doi.org/10.1016/0094-114X(94)90074-4).
- [2] C. Gologlu and M. Zeyveli, "A Genetic Approach to Automate Preliminary Design of Gear Drives," *Computers & Industrial Engineering*, vol. 57, no. 3, pp. 1043–1051, Oct. 2009, <https://doi.org/10.1016/j.cie.2009.04.006>.
- [3] V. Savsani, R. V. Rao, and D. P. Vakharia, "Optimal Weight Design of a Gear Train using Particle Swarm Optimization and Simulated Annealing Algorithms," *Mechanism and Machine Theory*, vol. 45, no. 3, pp. 531–541, Mar. 2010, <https://doi.org/10.1016/j.mechmachtheory.2009.10.010>.
- [4] M. Patil, P. Ramkumar, and K. Shankar, "Multi-Objective Optimization of Spur Gearbox with Inclusion of Tribological Aspects," *Journal of Friction and Wear*, vol. 38, no. 6, pp. 430–436, Nov. 2017, <https://doi.org/10.3103/S1068366617060101>.
- [5] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, Apr. 2002, <https://doi.org/10.1109/4235.996017>.
- [6] A. Konak, D. W. Coit, and A. E. Smith, "Multi-objective optimization using genetic algorithms: A tutorial," *Reliability Engineering & System Safety*, vol. 91, no. 9, pp. 992–1007, Sept. 2006, <https://doi.org/10.1016/j.res.2005.11.018>.
- [7] M. Mendez, D. A. Rossit, B. Gonzalez, and M. Frutos, "Proposal and Comparative Study of Evolutionary Algorithms for Optimum Design of a Gear System," *IEEE Access*, vol. 8, pp. 3482–3497, 2020, <https://doi.org/10.1109/ACCESS.2019.2962906>.
- [8] K. Deb and S. Jain, "Multi-Speed Gearbox Design using Multi-Objective Evolutionary Algorithms," *Journal of Mechanical Design*, vol. 125, no. 3, pp. 609–619, Sept. 2003, <https://doi.org/10.1115/1.1596242>.
- [9] M. Patil, P. Ramkumar, and S. Krishnapillai, "Multi-Objective Optimization of Two Stage Spur Gearbox using NSGA-II," in *International Conference on Advances in Design, Materials, Manufacturing and Surface Engineering for Mobility*, Chennai, India, July 2017, pp. 2017–28–1939, <https://doi.org/10.4271/2017-28-1939>.
- [10] E. S. Maputi and R. Arora, "Multi-objective Optimization of a 2-stage Spur Gearbox using NSGA-II and Decision-making Methods," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 42, no. 9, Sept. 2020, Art. no. 477, <https://doi.org/10.1007/s40430-020-02557-2>.
- [11] Q. Yao, "Multi-Objective Optimization Design of Spur Gear Based on NSGA-II and Decision Making," *Advances in Mechanical Engineering*, vol. 11, no. 3, Mar. 2019, Art. no. 1687814018824936, <https://doi.org/10.1177/1687814018824936>.
- [12] X.-H. Le and N.-P. Vu, "Multi-Objective Optimization of a Two-Stage Helical Gearbox using Taguchi Method and Grey Relational Analysis," *Applied Sciences*, vol. 13, no. 13, Jun. 2023, Art. no. 7601, <https://doi.org/10.3390/app13137601>.
- [13] L. D. Bao, V. D. Binh, D. V. Thanh, K. M. Nguyen, and L. X. Hung, "Multi-Objective Optimization of a Two-stage Helical Gearbox with Double Gears in the First Stage using MARCOS," *Engineering, Technology & Applied Science Research*, vol. 14, no. 6, pp. 18245–18251, Dec. 2024, <https://doi.org/10.48084/etasr.8865>.
- [14] T. Q. Hung, V. D. Binh, D. V. Thanh, L. A. Tung, and N. K. Tuan, "Multi-Objective Optimization of a Two-Stage Helical Gearbox with Two Gear Sets in First Stage to Reduce Volume and Enhance Efficiency using the EAMR Technique," *Engineering, Technology & Applied Science Research*, vol. 15, no. 1, pp. 19288–19294, Feb. 2025, <https://doi.org/10.48084/etasr.9224>.
- [15] Y. Lei, L. Hou, Y. Fu, J. Hu, and W. Chen, "Research on Vibration and Noise Reduction of Electric Bus Gearbox Based on Multi-objective Optimization," *Applied Acoustics*, vol. 158, Jan. 2020, Art. no. 107037, <https://doi.org/10.1016/j.apacoust.2019.107037>.
- [16] L. Qi, J. Zhou, and H. Xu, "Multi-Objective Optimization of Gearbox Based on Panel Acoustic Participation and Response Surface Methodology," *Journal of Low Frequency Noise, Vibration and Active*

- Control*, vol. 41, no. 3, pp. 1108–1130, Sept. 2022, <https://doi.org/10.1177/14613484221091075>.
- [17] L. Chat and L. V. Uyen, *Design and Calculation of Mechanical Transmissions Systems*, vol. 1. Hanoi, Vietnam: Educational Republishing House, 2007.
- [18] D. Jelaska, *Gears and Gear Drives*, 1st ed. Hoboken, NJ, USA. Wiley, 2012.
- [19] S. Kusumadewi, S. Hartati, A. Harjoko, and R. Wardoyo, *Fuzzy Multi-Attribute Decision Making (Fuzzy MADM)*. Yogyakarta, Indonesia: Graha Ilmu, 2006.
- [20] Ž. Stević, D. Pamučar, A. Puška, and P. Chatterjee, "Sustainable Supplier Selection in Healthcare Industries using a New Mcdm Method: Measurement of Alternatives and Ranking According to Compromise Solution (MARCOS)," *Computers & Industrial Engineering*, vol. 140, Feb. 2020, Art. no. 106231, <https://doi.org/10.1016/j.cie.2019.106231>.