

# Safety Index Evaluation and Calibration of Load and Resistance Factors for Concrete Beams Under the Simultaneous Effects of Bending, Shear and Torsion

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**Abstract**—The conventional approach for designing reinforced concrete members is based on load and resistance factors. In spite of the fact that, the load and resistance parameters are random variables, constant values are designated to them during the design. However, accounting these factors as constants, will ultimately lead to unsafe and uneconomical designs. Safe designs of structures require the appropriate recognition of parameters and their uncertainties. This may be achieved by clarifying the effective design parameters and applying risk-based design methods. The main purpose of this paper is the reliability-based design of reinforcement concrete beams under the simultaneous effects of bending, shear and torsion. For this purpose, a computer program is developed, and the most usual sections are selected for studying. Rectangular sections with tension reinforcement (singly reinforced), rectangular sections with tension and compression rebars (doubly reinforced) and T-shape sections are designed based on probabilistic methods. An appropriate tool for reliability calculations is selected based on the comparisons of different methods. Evaluation of load and resistance factors for all mentioned beams is conducted. The steel stresses under the safety levels are determined. Hence, a method for the simultaneous and fully probabilistic design of concrete beams is proposed.

**Keywords**—reinforced concrete beams; reliability-based design; monte-carlo simulation; safety factor; load and resistance factors.

## I. INTRODUCTION

Using new designing methods based on reliability is a relatively old issue in civil engineering [1] and a lot of researches on reliability based design have been conducted because the existing uncertainties in designing parameters and the use of constants in regulations sometimes overshadow the design. So, using reliability based methods based on the limit state governing the problem and paying attention to statistical parameters for each designing variable such as mean, standard deviation and standard distribution. Various regulations of designing have been followed in various countries. For example, in the shear force of reinforced concrete for a reinforced concrete beam, regulations using a new reliability method are calculated. It has been indicated that various factors are effective in the failure of concrete beam including the ratio of the length over the effective depth,

the ratio of reinforcement steel, support conditions, loading conditions, and the type of materials. In (3), authors investigated the reliability of a concrete beam. The studied beam was designed under the effect of shear-bending-torsion. The failure level for three states of shear, bending, and torsion was considered according to the first order reliability method (FORM) and indicated the fact that the safety index is dependent to the rate of live load, the rate of effective plastic moment, beam strength as well as the uncertainty of the model.

In [5], authors studied risk management in the reliability of concrete infrastructures. In [6], authors investigated the reliability indices in beams and columns obtained for ACI-regulation and the designed structures. In [7], authors studied the punching shear using the reliability method in a flat slab. In [8], a research in order to obtain the reliability index to show shear and reinforcement of bridges concrete was carried out. In [9], authors studied the reliability of a concrete bridge in which the intended beam was under the effect of moment and shear forces. In [10], the concrete safety index was investigated with regard to uncertainty parameters in various material. In [11], Greek seismic regulations were investigated focusing at the calibration of the regulations for the beam-column and beam members. In [12], authors studied the impact of the calibration of regulations and the theory of probability in designing concrete bridges performance.

In accordance with the recommendations made in [4] and other statistical studies carried out by researchers such as [13, 14], the safety index for dead load + live load is 3 for bending and 2 for shear and torsion that in the present study the interval of the safety index to the proposed value is studied. The aim of this study is to investigate the safety index of Iranian concrete regulations for three limit states of shear, bending and torsion and also for the combination of the above limit states. The present study, suggesting the limit relations, has investigated the limit functions of regulation and has calculated the safety level and coefficients for all proposed states.

According to these studies, in the present paper after determining the factors of uncertainty in a beam affected by torsion and based on effective limit functions in designing, the

safety level of Iranian concrete regulations has been estimated using the Monte-Carlo method. To achieve this goal, the existing parameters of uncertainty have been recognized and its random production method is with a high number of about 1500000 and is based on statistical parameters such as mean, standard deviation and statistical distributions. Then, various limit states such as shear, torsion and bending have been considered and the safety level and the factors addressed in the regulations have been estimated.

II. DESIGNING OF CONCRETE BEAMS UNDER BENDING, SHEAR AND TORSION CONDITIONS

In order to calculate the safety index, we should calculate the limiter functions which show the structural efforts. The relations of bending calculating, shear and torsional efforts are presented in the following. The resistant moment of the section is given in (1), the shear force of the section in (2) and the torsion moment of the section in (3).

$$M_r = A_s f_y \left( d - \frac{A_s f_y}{1.7 f_c b d} \right) \tag{1}$$

$$V_r = 0.2 \phi_c b_w d \sqrt{f_c} + \phi_s f_y A_v \frac{d}{s} \tag{2}$$

$$T_r = 2 \phi_s A_o A_t \frac{f_y}{s} \tag{3}$$

In (1) to (3),  $M_r$  is the section moment of resistance,  $V_r$  is the shear moment of resistance and  $T_r$  is the torsional moment of resistance. Also, the parameters of  $A_s$ ,  $f_y$ ,  $f_c$ ,  $b$  and  $d$  are the area of steel, tensile strength of the steel used, the concrete compressive strength, width of section, the concrete compressive strength, tensile strength of the steel used and the amount of steel required in the section, respectively. Using (1) to (3) separately for the states of shear, bending and torsion based on forces applied on the sections, we have calculated the minimal values of the used steel with regulation [15]. In the case of bending design in all sections, the compressive strength is used. In the case of L-shape and T-shape sections, the proposed equations have been used by regulations to obtain the section moment of resistance, and all used equations are available in [15]. For shear and torsional states of beam, the full sections of the beam (L, T and rectangular) have been considered. The factors considered in the concrete regulations of Iran to increase the load factor and reduce the resistance in designing are  $\gamma_D=1.25$ ,  $\gamma_L=1.50$  in limit state for the dead load and live load respectively. As well as the reduction factors of steel and concrete strengths in design limit state are  $\phi_s=0.85$  and 0.60 respectively. The needed statistical information to analyze the reliability are provided in Table 1.

III. FORMULATION OF THE LIMITER FUNCTION AND SAFETY INDEX

For designing all possibilities of concrete beams under the simultaneous effects of bending, shear and torsion, the limit functions must be considered in the following conditions:

$$\begin{aligned} M_r &\leq M_s \\ V_r &\leq V_s \\ T_r &\leq T_s \end{aligned} \tag{4}$$

In (4), the limit value are the forces that a beam can tolerate. To estimate the safety index, first the amount of steel required of section for three mentioned forces is calculated and then using Hasofer-Lind equation [16] the safety index is calculated according to (5).

$$\beta = \frac{\mu - \frac{\sigma}{\sqrt{2}}}{\sigma} \tag{5}$$

In this study, the safety index is studied for different limit states in the concrete beams from different aspects using Monte Carlo technique.

TABLE 1. STATISTICAL PROFILE OF THE PARAMETERS USED IN THE RESEARCH FOR DESIGNING THE SECTIONS OF REINFORCED CONCRETE (13-14)

Random variable	Number of value	Density Function	Mean	Standard deviation
$f_c$ (MPa)	21	Normal	19.3	0.18
$f_y$ (MPa)	420	Normal	317	0.12
Section	b	Normal	b	b/10
	h		h	h/17
	d		d	d/15
Area (mm <sup>2</sup> )	$A_s$	Normal	$A_s$	0.03
	$A_v$		$A_v$	0.03
	$A_t$		$A_t$	0.03
Loading	D	Normal Gamble	1.05D	0.1
	L			0.40-0.25

Safety level of shear-torsion

Based on the previous studies, by limiting the equation (6) to a specified value, the combination of simultaneous effect of shear-torsion can be achieved in various sections of concrete.

$$\frac{v_r}{b_w d} + \frac{2T_u (b_w + d)}{(b_w d)^2} < 0.25 \phi_c f_c \tag{6}$$

The safety level estimation of Iranian concrete regulations for shear and torsion is as follows: The amount of the shear steel is calculated according to the considered section and also (2). The calculating method is such that by assuming the certain amount of the dead load and placing the parameter ratio=(1-t)/t and changing the value of t from 0.4 to 1, the amount of live load gets calculated. The amount of shear load is obtained from adding the live load to the dead load and finally, with respect to (2) the required amount of shear steel is obtained. Applying the value of the moments acting on the beam (the moment of dead load + the moment of live load) and (3), the required torsional steel is calculated. The method of calculating the total moments exerted on the beam is similar to those of shear state and is practical by considering the proposed ratio in [3] for the amount of t. The limiting function considered for solve the problem by take the formulas presented in regulations for shear, torsion and the combination of shear and torsion is as in (7) to (9), respectively.

$$G = \frac{v_r}{b_w d} - \frac{v_s}{b_w d} \tag{7}$$

$$G = \frac{2T_r(b_w+d)}{(b_w d)^2} - \frac{2T_s(b_w+d)}{(b_w d)^2} \tag{8}$$

$$G = \left( \frac{v_r}{b_w d} + \frac{2T_u(b_w+d)}{(b_w d)^2} \right) - \left( \frac{v_s}{b_w d} + \frac{2T_s(b_w+d)}{(b_w d)^2} \right) \tag{9}$$

In the above equations, the index S is the load and moment exerted on the section (demand), r is the resistance level of the section (capacity) against acting loads. In sections 3-2 to 6-2, different levels of regulations are presented for various combinations of forces.

**B. The safety level of bending-torsion**

To investigate the safety level of Iranian regulations, the amount of shear and bending steel is calculated using (1) to (3). After calculating the amount of the moment of resistance exerted on the section and the amount of the applied loads, a limiting function such as in (10) and (11) is considered.

$$M_r = A_s f_y \left( d - \frac{A_s f_y}{1.7 f_c b d} \right) - M_s \tag{10}$$

$$G = \left( \frac{M_r c}{I} + \frac{2T_r(b_w+d)}{(b_w d)^2} \right) - \left( \frac{M_s c}{I} + \frac{2T_s(b_w+d)}{(b_w d)^2} \right) \tag{11}$$

In the equations above, the effects resulted from the loads (live+dead) is shown with index S, and the resistance of the section against the exerted loads according to the characteristics of the constructed section is shown by index r.

**C. The safety level of bending-shear**

Previously the amount of the required steel has been calculated. Also, the amount of the forces acting on the section and the strength of the section has been obtained for the modes of shear, torsion and bending. For determining the limiting function for the bending and shear, the following limiting function is considered.

$$G = \left( \frac{v_r}{b_w d} + \frac{M_r c}{I} \right) - \left( \frac{v_s}{b_w d} + \frac{M_s c}{I} \right) \tag{12}$$

**D. The safety level of bending-torsion-shear**

To estimate the safety level of Iranian regulations for the three modes of shear, bending and torsion, the following limiting function are considered.

$$G = \left( \frac{M_r c}{I} + \frac{2T_u(b_w+d)}{(b_w d)^2} + \frac{v_r}{b_w d} \right) - \left( \frac{M_s c}{I} + \frac{2T_s(b_w+d)}{(b_w d)^2} + \frac{v_s}{b_w d} \right) \tag{13}$$

In the limiting function above, the effects of shear, bending and torsion loads are observed simultaneously and by considering the limiting function above the safety level was estimated. The investigation of the effect of dead load on the safety level, the value of dead load was changed.

**IV. RESULTS**

To investigate the safety index of concrete beams, a section with given geometrical specifications was considered as shown in Figure 1. The extensive load of 25kN/m was applied on the beam. The live load is 5kN/m and the beam sections were L-shape, T-shape and rectangular. Choosing the beam was in such a way that the section area of the T-shape beam was two times bigger than the section area of the L-shape beam and for rectangular section was as d\*b<sub>w</sub>. For each case, the total section and designing tables have been presented. Designing of the beam, at first has been done in accordance with Iranian concrete regul...

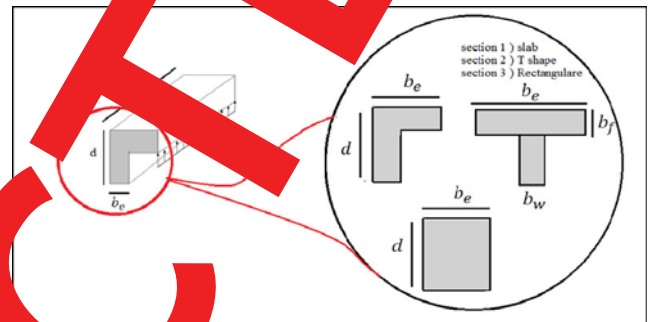


Figure 1. Cross-sections of materials and dimensions considered for the studied sections

TABLE II. THE NOMINAL USED VALUES

b <sub>w</sub> (mm)	q <sub>0</sub> (kN/m)	f <sub>c</sub> (MPa)	B <sub>e</sub> (mm)	f <sub>y</sub> (MPa)
400	25	21	800	420
b <sub>r</sub> (mm)	d (mm)	L (mm)	M <sub>1</sub> =ratio*M <sub>D</sub> =(1-t)/t	
200	400	2000	0.4-1	

**The safety level of regulations for states of torsion-shear**

The safety index has been calculated considering the limiting function and statistical values given in Table I. The calculation method is as follows. Nominal values such as the dimensions of the beam, the amount of shear-torsional steel, the amounts of exerted loads on the section, the concrete compressive stress and the tension of steel fluidity and etc., were considered as random variables according to Table I. Also, using the statistical values obtained such as the mean, standard deviation and statistical distribution, we have calculated the exact amount of shear force acting on the beam and the moment of torsion. The method of estimating the values of the moment of torsion and the tolerable shear force of the section was the extensive production of random numbers per each cycle written in MATLAB [16]. The Monte Carlo Method was used in procedures, which at the beginning of the cycle the values of each variable for each cycle of production, the amount of the moment and resistance (R) were calculated and at the end of each cycle the amount of load exerted on the section (S), the total load acted on the section and the moment of shear will be calculated. The safety indices in the section for various load ratios are estimated using (5).

In Figure 2, the impact numbers of Monte Carlo periodic cycles on the numerical value of the safety index are shown. This figure shows the safety index of Iranian concrete regulations for torsion and shear per different speeds. The considered section to investigate the number of Monte Carlo cycles is rectangular.

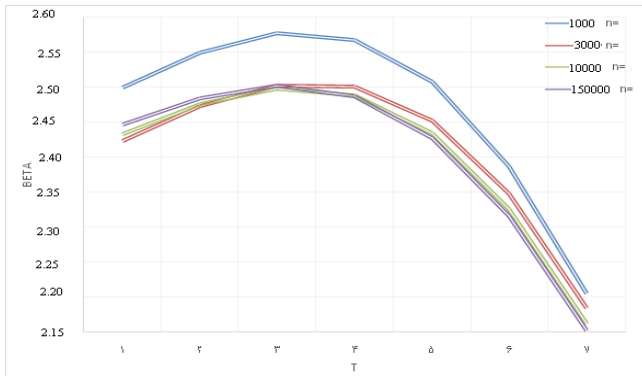


Fig. 2. Safety level

By considering Figure 2, it is clear that for all load values the recommended safety level of Iranian concrete regulations is higher than the suggested number of 2 for shear and torsion (20). Safety index reaches its maximum in number of 0.6. Also, speeding up the cycle to the number of 15000 estimates the safety level with a good approximation and for higher speeds the safety index showed little changes. As is seen in the figure, the calculations for the number of rounds to 10 times, i.e., 1000, 3000, 10000, 150,000 rounds have been done, but the safety index has not changed much. So, in follow-up research the number of simulated rounds has been limited to 15,000. As can be seen from the figure, reducing the live load (increasing the numerical value of  $t$ ) has led to a decline in the safety index that is likely because of the loss of influence of the safety factor (factor of 1.5).

**B. The safety level of bending-torsion**

Using the formulas presented earlier, the safety index is calculated for loading combination of bending-torsion. In order to present and compare the results, the bending levels have been shown with separated lines and the torsion levels with dotted lines and the combination of the considered states with continuous lines; the rectangular sections are shown with blue color, L-shape section with red color, and T-shape section with green color. In the graph shown below the safety index for each section has been presented.

According to Figure 3, it is clear that the regulation level of Iranian concrete for torsion is from 2.40 to 2.70, and the average obtained is 2.48. Also, the regulation level of Iranian concrete for bending is between 3 and 4 and the average obtained is 3.05. The regulation level of Iranian concrete is for bending and torsion is between 2.90 and 3.50, and the average obtained is 2.90 for bending-torsion. The maximum of safety index for combination of shear-bending occurs at  $t=0.6$ .

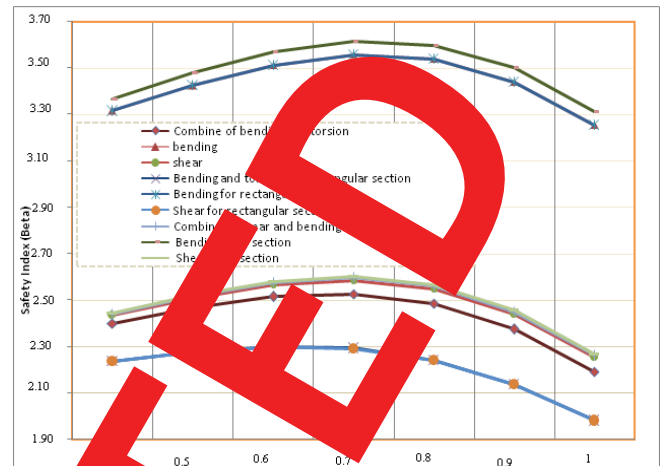


Fig. 3. Regulation level of Iranian concrete for bending-torsion

**C. Safety level of bending-shear state**

Figure 4 shows the regulation safety level of Iranian concrete for shear-bending. Figure 4 also shows the regulation level for limit state of shear-bending. The regulation level in the combination of two limit states will skew towards shear. As seen in the figure, the regulation level at shear and bending are not far from each other. Shear and bending levels and the combination of the two states have been shown with dotted points, dotted lines and line respectively. To present and compare the results better the results of shear levels have been shown with dotted points, the combination of the considered states has been shown with lines, and the rectangular sections have been shown with blue color, the slab section with red color, and T-shape section with green color.

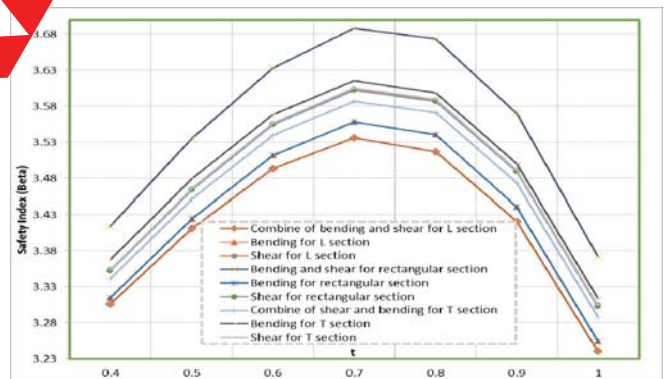


Fig. 4. Regulation of the fathers to the Sun-Bending

**D. The safety level of bending-torsion-shear**

Combination of the shear, bending and torsional states has been done using existing formulas. Figure 5 shows the maximum safety index for the three states above related to T-shaped section that is 3.05

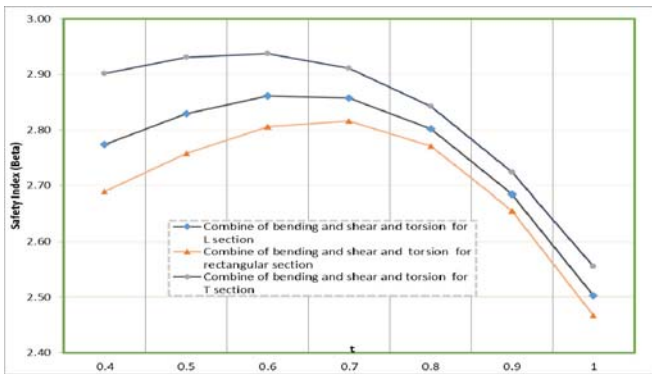


Fig. 5. Regulation level of Iranian concrete for shear, bending and torsion

E. Estimation of the regulation safety factors for limit states. Safety factors calculated with the safety index (shear-torsion-bending)

In the following section, the factors of regulations are estimated. In order to estimate the factors, only the data of the rectangular section have been used. To obtain the safety factor, at first a wide range of factors are used. Then, using the existing formulas in the Iranian concrete regulations and the applied factors, the nominal values of steel are calculated. Next, the safety index of the considered limit functions are calculated without the use of any of the factors existed in the regulations and by using Monte Carlo cycle. In Figures 5 to 8, the coefficients obtained from the Monte Carlo technique and the intended safety index are presented. As it can be seen from the figures, the estimated safety factors have a direct influence on the safety index and they affect the fluctuations of the safety index. Also, it can be seen that the increase of the amount of live load and dead load, and consequently reducing the resistance reduction factors (steel, concrete) reduces the safety index while the increase is linear.

Figure 8 illustrates this important issue that at the states of shear and torsion, Iranian concrete regulations are close to torsion and at the states of torsion and bending, Iranian concrete regulations are near to torsion. As can be seen in figure 8, the increase of resistance reduction factor ( $\phi_{cs}$ ) reduces the safety index, because the safety index is directly affected by loads than resistance reduction factors, and the more the effect of the load, the higher the safety level; Therefore, to investigate the importance of these factors in the safety index, the increase of resistance factors (that leads to a decrease in safety index) and the increase of load factors (that leads to an increase in safety index) have been studied and the safety levels and the effectivity from the factors are compared and discussed.

Figures 6, 7 and 8 indicate that with respect to regulation formulas, the effective factors in safety index at states of shear and torsion is more effective than load factors and on the contrary, at bending state the Iranian regulation level concrete is more affected by resistance reduction factors; and at the combination of torsion-bending (the index is more affected by torsion) compared with the state of shear and bending (the index is more affected by shear), the regulation level is more affected by the increase of resistance reduction

factors and the increase in load at torsional state could not affect the safety index and the resistance factors have been more effective on safety index at state of bending-torsion. The slopes of drawn lines are in inverse order.



Fig. 6. Estimation of the factors used in Iranian concrete regulations for shear-bending

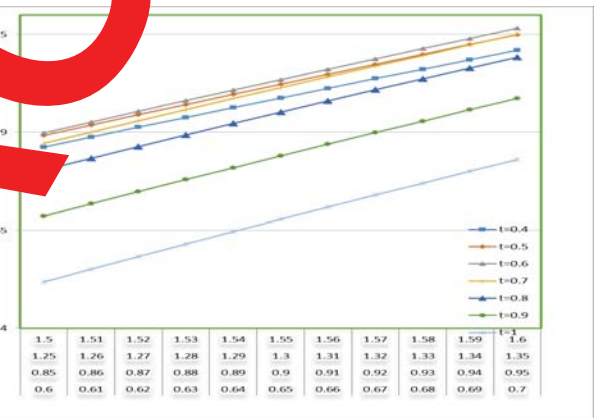


Fig. 7. Estimation of the factors used in Iranian concrete regulations for torsion-shear

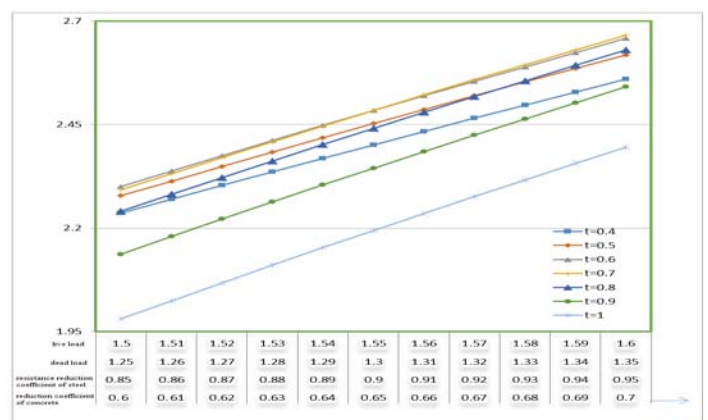


Fig. 8. Estimation of the factors used in Iranian concrete regulations for bending-torsion

The following conclusions can be obtained:

- At bending state and its combination with torsion, the regulation level is more affected by resistance reduction factors and the increase of these factors decreases the safety index further.
- According to the dimensions and load given, the regulation level is affected by shear steel. Certainly, the change of dimensions leads to the changes in used steel and the rate of affectability from shear because the size and composition of load directly affect the safety level.
- The regulation level of Iranian concrete at general state of the combination of bending-torsion-shear at  $t=0.7, 0.9, \&1$  is affected by the increasing of load factors and at  $t=0.4-7$  the regulation level is affected by resistance reduction factors; therefore the increase of live load to dead load directly affects the safety level in all regulation states.
- For an increase of the amount of  $t=0.7, 0.9, \&1$  in the state, the shear is effective and consequently the increase in load factors have more power for increasing the safety index, because in these states the increase of the effect of the live load factor has affected the safety level.
- The absolute values of the slope of the drawn lines are the power of the factors in changing the numerical value of the safety index.

#### V. CONCLUSION

This study investigated the safety level of Iranian concrete regulations for limited combinations of torsion-shear-bending and obtained the regulation safety level of Iranian design using the Monte Carlo technique. The basic results are as follows:

- The safety index obtained for bending, shear, and torsional state is set between 2 and 3 (considering that in shear state the safety level is higher than at the bending state, and in the bending state it is higher than at the torsion state).
- The combinations of limit state for shear, torsion and bending are near to the safety index of shear and in load combinations reaches to its maximum level which indicates that the increase level of the live load factor and dead load factor are at their maximum level.
- An increase in the section area will increase the safety level of regulations; because at the amount of fixed load the increase in concrete results to the reduction of resistance factors that directly affect the safety index.
- At all calculated load factors the regulation level is set higher than the recommended safety index, which indicates that factors entering in regulation result to an unsafe design. Certainly, changing of the dimensions of the beam and also the rate of exerted loads will affect the safety index. Therefore, a more attention regulation principles results in changing the safety in different conditions; so the accurate design should be done with careful study and clarification of the ultimate goals according to the lifetime of the considered structure.

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