Optimization of NURBS Parameters for Computing Kinematic Characteristics of the Cam Oscillating Roller Follower

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ABSTRACT

This paper presented an approach for the design and optimization of cam mechanisms with oscillating roller followers using Non-Uniform Rational B-Spline (NURBS) curves. The follower's motion and its kinematic parameters, including velocity, accelerations, and jerk were expressed through a NURBS transfer function. To optimize the problem, the maximum values of acceleration and jerk were minimized. Additionally, a simulated annealing algorithm was developed to determine the optimal NURBS parameters. An example in cutting machines was used to evaluate this method individually and comparatively with other two methods (B-Spline and polynomial). The results demonstrated that the proposed method provided flexibility in using the transfer function for the follower in order to improve the kinematic characteristics of the cam systems. This research not only raises the significance in the design of cam oscillating roller follower, but also hands out the potential application for other related cam systems.

Keywords-cam optimization; kinematic characteristics; NURBS curves; transfer function

I. INTRODUCTION

Cam mechanisms consist of two kinematic elements: the cam and the follower. They are flexible, consistent, and economical in creating a movement with the desired requirements. One of the main advantages of cam mechanisms is their ability to produce arbitrarily specified output follower motion with an accurate tight control, due to their kinematic characteristics. Furthermore, they provide mechanical efficiency due to the reduction in the number of motions involved in such motions.

Cam-follower systems are widely used in many machine applications, including internal combustion engines (to control valves), automatic machinery, and cutting tools. In the design process of cam mechanisms, transfer functions play a crucial role as they determine the kinematic and dynamic behavior of the system. These functions affect the velocity, acceleration and jerk of the follower. Standard functions including harmonic, cycloidal, trapezoidal, and polynomial have been utilized for the displacement motion [1-3]. In addition, many studies have proposed spline-based functions for describing the follower motion [4-10].

Regarding the optimization problem in cam design, authors in [11] examined the optimization of cam follower systems

with kinematic and dynamic characteristics, using the polynomial function for the motion curves. Additionally, authors in [12] presented an optimum design of cam mechanisms with a translating flat-faced follower. A cam mechanism in internal combustion engine has also been investigated using the Finite Element Method (FEM) [13]. In [14], a cam profile design optimization model was proposed based on a sixth-order spline to minimize vibrations in single degree of freedom cam systems. Another study explored the optimization of cam follower using the B-spline function to minimize the peak value of the jerk [15].

Due to the flexibility and simple structure of the cam mechanisms, cam oscillating roller followers are one of the most popular types among others in the mechanical systems. Several parameters affect their kinematic and dynamic performance at the design stage, namely radius of follower roller, radius of base circle, eccentricity, and follower transfer function.

This paper emphasizes the optimization of the transfer function of the follower considering cam mechanism kinematic characteristics with a roller follower. An NURBS curve is used to model the displacement function of the follower, where the objective function is based on the peak value of acceleration and jerk during the system operation; these good designs

include the generation of a cam profile with a reasonable pressure angle. The problem of optimization and deduction of optimal NURBS parameters have been solved by the simulated annealing optimization algorithm.

II. OPTIMIZATION STATMENT

This section introduces the geometry of cam mechanisms with oscillating roller followers. The follower's transfer function and its kinematic characteristics are proposed by the NURBS curves. The objective function, expressed for the dynamic properties, is constructed based on this transfer function. To solve the optimal problem, a simulated annealing algorithm is employed.

A. Description of the Cam Oscillating Roller Follower

A cam mechanism with an oscillating roller follower is illustrated in Figure 1. The transfer function of the system is defined as a function of θ , and can be expressed as:

$$y(\theta) = \sum_{i=1}^{n} R_{i,p}(\theta) P_i$$
, for $\theta \in [a, b], i = 1, 2, ..., n$ (1)

where $y(\theta)$ is the angular displacement of the follower measured in radians, Pi are the control points, p is the degree of the NURBS function, θ is the camshaft angle (in radians), $[a,b] \subseteq [0,2\pi]$ is the defined angular range, and $R_{i,p}(\theta)$ are the rational basis functions.

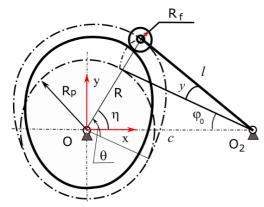


Fig. 1. Diagram description of cam oscillating roller follower.

The kinematic characteristics of the cam mechanisms are computed by:

$$y'(\theta) = \frac{dy(\theta)}{d\theta} = \sum_{i=1}^{n} R_{i,p}^{1}(\theta) P_{i}$$
 (2)

$$y''(\theta) = \frac{d^2y(\theta)}{d\theta^2} = \sum_{i=1}^n R_{i,p}^2(\theta) P_i$$
 (3)

$$y^{\prime\prime\prime}(\theta) = \frac{d^3y(\theta)}{d\theta^3} = \sum_{i=1}^n R_{i,p}^3(\theta) P_i \tag{4}$$

Here, $y'(\theta)$, $y''(\theta)$, and $y'''(\theta)$ represent the velocity, acceleration, and jerk, respectively and depend on the derivatives of the rotational basis functions $R_{i,p}(\theta)$:

$$R_{i,p}(\theta) = \frac{N_{i,p}(\theta)w_i}{\sum_{j=1}^n N_{j,p}(\theta)w_j}$$
 (5)

where w_i are the weight parameters which affect the shape of the NURBS curve.

The B-spline basis functions, $N_{i,p}(\theta)$ are expressed as:

$$N_{i,0}(\theta) = \begin{cases} 1, & \text{if} & \theta_i \le \theta \le \theta_{i+1} \\ 0, & \text{otherwise} \end{cases}$$
 (6)

$$N_{i,p}(\theta) = \frac{\theta - \theta_i}{\theta_{i+p} - \theta_i} N_{i,p-1}(\theta) + \frac{\theta_{i+p+1} - \theta}{\theta_{i+p+1} - \theta_{i+1}} N_{i+1,p-1}(\theta)$$
(7)

The knot vector θ is used to define the basis functions given by:

$$\theta = \{\underbrace{a, \dots, a}_{(p+1)}, \theta_{p+2}, \dots, \theta_{m-p-1}, \underbrace{b, \dots, b}_{(p+1)}\}$$
(8)

The derivatives of $R_{i,p}(\theta)$ are determined based on the derivatives of the basis functions as:

$$N_{i,p}^{k}(\theta) = \frac{p!}{(p-k)!} \sum_{j=0}^{k} a_{k,j} N_{i+j,p-k}(\theta)$$
 (9)

where:

$$a_{0,0} = 1$$

$$a_{k,0} = \frac{a_{k-1,0}}{\theta_{i+p-k+1} - \theta_i}$$

$$a_{k,j} = \frac{a_{k-1,j} - a_{k-1,j-1}}{\theta_{i+p-k+1} - \theta_{i+j}}$$

$$a_{k,k} = \frac{-a_{k-1,k-1}}{\theta_{i+p+1} - \theta_{i+k}}$$
(10)

Using these formulations, the kinematic characteristics of the cam follower mechanism can be determined.

B. Objective Function

In the operation of cam systems, it is important to consider the effects of dynamic forces and vibrations. To achieve this, the peak values of acceleration and jerk should be minimized. The NURBS function is flexible in controlling curves since its parameters can be dominated by this optimization. Thus, the objective functions can be expressed as:

Minimize
$$f(\mathbf{X}, \mathbf{Y}) = [f_1(\mathbf{X}, \mathbf{Y}), f_2(\mathbf{X}, \mathbf{Y})]^T$$
 (11)

where, $f_I(X, Y)$, $f_2(X, Y)$ are the maximum acceleration and jerk functions, respectively, which depend on NURBS weight and knot values. Based on (3) and (4), these values can be expressed as:

$$f_1(X,Y) = \max y''(w_1, w_2, \dots w_i, \theta_{n+2}, \dots, \theta_{m-n-1})$$
 (12)

$$f_2(X, Y) = \max y'''(w_1, w_2, \dots w_i, \theta_{p+2}, \dots, \theta_{m-p-1})$$
 (13)

The decision variables X and Y are defined as:

$$X = [w_1, w_2, \dots w_i]$$
 (14)

$$\mathbf{Y} = [\theta_{p+2}, \theta_{p+3}, \dots, \theta_{m-p-1}]$$
 (15)

where i = 1, 2, ... n is the number of weights and m is the total number of knots.

The constraints for the objective function can be described as:

$$w_i > 0 \tag{16}$$

$$a < \theta_{n+2} < \dots < \theta_{m-n-1} < b \tag{17}$$

C. Simulated Annealing Algorithm for Solving the Optimal Problem

Simulated annealing is a stochastic method for solving the optimal problem. This process involves tempering some type of metal, glass, or crystal by heating it above a critical temperature, maintaining it for a time, and then gradually cooling it until it solidifies into a complete crystalline structure. The details of this procedure are analyzed in [16]. The simulated annealing algorithm has been widely utilized in various engineering problems, such as damping vibration optimization [17], or location in distribution networks with embedded wind power generation systems [18]. In this study, this algorithm was used to find the optimal NURBS parameters in order to minimize the nonlinear function. The steps for solving the optimal problem are presented in Figure 2.

```
Begin
             X_0, Y_0: Initial population of the variables
             f_0 = f(X_0, Y_0): Objective function at the initial population
             To: Initial control parameter
            α: Cooling rate
             Tmin: Termination criterion
Repeat
   Repeat
             X_i = X_0; Y_i = Y_0 (X_i, Y_i: Current solution)
             f_i = f(X_i, Y_i) (Objective function at current solution)
            \pmb{X}_{i+1} = \pmb{X}_i + \Delta \pmb{X} ; \pmb{Y}_{i+1} = \pmb{Y}_i + \Delta \pmb{Y} (Generate randomly a
neighboring solution)
            f_{i+1} = f(X_{i+1}, Y_{i+1}) (Objective function at the neighboring
solution)
             \Delta f = f_{i+1} - f_i
            If \Delta f \leq 0 then
                         X_i = X_{i+1}; \ Y_i = Y_{i+1}
            Else
                         If random(0,1) < p
                                      X_i = X_{i+1}; Y_i = Y_{i+1}
                                       Return to generate the new solution
                         End
            End
    End
             Set T_{i+1} = \alpha T_i (Decrease the control parameter)
Until
            Termination criterion T \leq T_{min}
End
```

Fig. 2. Simulated ammealing algorithm for solving the NURBS parameters.

III. COMPUTATION OF CAM PROFILE

The cam profile is calculated in the Cartesian coordinate system using an inversion method, as depicted in Figure 1. The cam profile can be computed using:

$$x_c = c \cos\theta - l \sin(\frac{\pi}{2} - \theta - \varphi_0 - y)$$
 (18)

$$y_c = -c \sin\theta + l \cos(\frac{\pi}{2} - \theta - \varphi_0 - y)$$
 (19)

where:

$$\varphi_0 = a\cos\frac{(c^2 + l^2 - R_p^2)}{2lc}$$
 (20)

The pressure angle, denoted by α , can be calculated by:

$$\alpha = sig\left[\frac{\pi}{2} - asin\left(\frac{c}{R}\right)\right] + atan\left[\frac{1}{\frac{R^2}{|c\sin(\varphi_0 + y)y'|^2} \frac{c^2 - R^2 - |z|^2}{2Rc\sin\eta}}\right] (21)$$

where, sig = +1 for the rise period, and -1 for the return period. R and η can be calculated by:

$$R^2 = x_c^2 + y_c^2 \tag{22}$$

$$\eta = a\cos\left[\frac{c^2 + R^2 - l^2}{2Rc}\right] \tag{23}$$

IV. APPLICATION EXAMPLE

In this section, a cam design example for a cutting machine is presented [19]. The machine utilizes a cam mechanism with an oscillating roller follower. For this cam, three periods were considered: rise, dwell, and return.

In the rise motion, the follower satisfies ten positions of the cutting tool for the working cycle of the machine. The rise motion is defined by boundary conditions, including velocity, acceleration, and jerk at the start and end of the rise. For the other points, the same parameters do not require exact values and are signed by "--". The boundary conditions of the cam oscillating roller follower are outlined in Table I. In the return motion, the cutting tool must go backward fast. The movement needs to satisfy the boundary conditions at the start and end of the motion curves, leading to smooth displacement, velocity, and acceleration. Therefore, the boundary conditions for this period at the first and end points of displacement are also detailed in Table I.

TABLE I. BOUNDARY CONDITIONS OF CAM OSCILLATING ROLLER FOLLOWER USED IN CUTTING MACHINE EXAMPLE

Angle of camshaft (rad)	Dispacement (mm)	Velocity (rad/s)	Accelaration (rad/s²)	Jerk (rad/s³)
Boundary conditions for rise period				
0	0	0	0	0
0.3490	0.0176			
0.6981	0.0669			
1.0471	0.1391			
1.3961	0.2218			
1.7455	0.3022	1		-
2.0945	0.3684	-		
2.4435	0.4125			
2.7926	0.4328			
3.1416	0.4363	0	0	0
Boundary conditions for return period				
3.1416	0.4363	0	0	0
4.7124	0	0	0	0

The application of the simulated annealing algorithm, as displayed in Figure 2, is employed for solving the objective function (11). The objective was to minimize the maximum acceleration and jerk by optimizing the NURBS parameters.

The optimal NURBS parameters for the rise motion are:

X = [0.3275, 0.6153, 0.9940, 1.3475, 1.0819, 0.9145, 0.9405, 1.1660, 0.9677, 1.0555, 0.7493, 1.1279, 1.0132, 0.7671, 1.0885, 1.0667],

 $\mathbf{Y} = [0.2094, 0.4188, 0.6981, 1.0472, 1.3962, 1.7453,$

2.0944, 2.4435, 2.4435, 2.7228, 2.9322].

The control points P_i of the NURBS function are obtained from the boundary conditions:

P = [0, 0, 0, 0, 0.0473, 0.0656, 0.1349, 0.2221, 0.3086, 0.3757, 0.4095, 0.4302, 0.4363, 0.4363, 0.4363, 0.4363].

The number of the control points is equal to the number of the weights. The vectors in the return period are:

X = [0.3199, 0.5508, 0.7300, 1.1581, 1.1498, 0.7696, 0.5183, 0.4170],

Y = [3.4749, 3.8083],

P = [0, 0, 0, 0, 1, 1, 1, 1].

Therefore, the transfer and kinematic functions of the follower were established using (1)-(4). The diagrams of these functions are presented in Figures 4-7. These functions were compared with the transfer function and its kinematics using the polynomial curve and B-spline function (Appendix). These functions are satisfied by the requirement of the boundary conditions. The coefficients of the polynomial functions in (24), $c_0, c_1, c_2, ..., c_n$ and the control points, P_i , of B-spline are determined by the boundary conditions.

Figure 3 illustrates the diagram of the transfer function of the follower used in cutting machine systems. It can be observed that the difference between the NURBS transfer function (blue line) and the polynomial function (red line) is very small. However, the velocity, acceleration, and jerk values from all methods are quite different. As for velocity, as shown in Figure 4, the maximum values are 0.2784 rad/s for NURBS, 0.2510 rad/s for B-spline, and 0.2076 rad/s for polynomial. In Figure 5, the maximum values of acceleration are 1.2197 rad/s² for NURBS, 1.5068 rad/s² for B-spline, and 1.3286 rad/s² for the polynomial curve. In addition, the maximum values of jerk are 4.7341 rad/s³ for NURBS, 7.5990 rad/s³ for B-spline, and 5.9104 rad/s³ for polynomial, as evidenced in Figure 6.

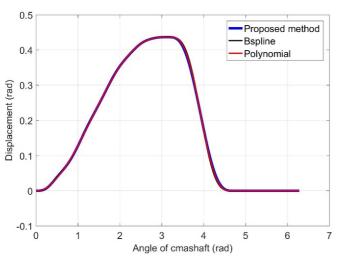


Fig. 3. Transfer function of the follower.

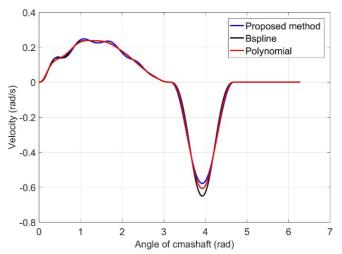


Fig. 4. Velocity of the follower.

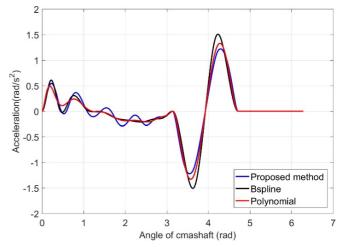


Fig. 5. Acceleration of the follower.

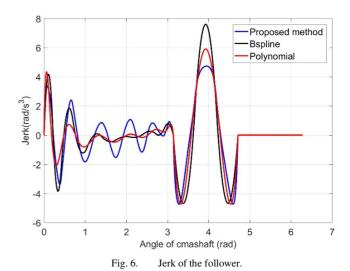
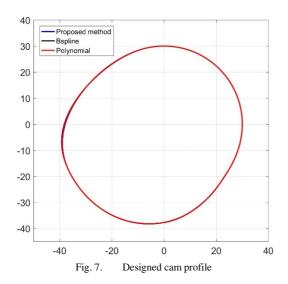


Figure 7 illustrates the cam profiles designed for the cam follower systems using three different curve methods: NURBS,

B-spline, and polynomial functions. These profiles were obtained from (18) and (19). It is clear that the cam profiles generated by all three methods were similar. However, the kinematic characteristics of the follower included different maximined values. Due to this, the transfer function played an important role in cam system design.

The pressure angle was calculated during the computation of the cam curve for both the rise and return motion using (21). The results confirmed that the pressure angle remained below 35°, which is within the acceptable limits [1].



V. CONCLUSION AND FURTHER WORK

This study presented the procedure for the computation and optimization of cam mechanisms with oscillating roller followers. The Non-Uniform Rational B-Spline (NURBS) curve was employed to describe the transfer function and its kinematic characteristics. Due to the flexibility of NURBS with weight and knot parameters, the kinematic characteristics were improved based on the optimization conditions. The simulated annealing algorithm was also applied to optimize these parameters. The comparative results revealed that the NURBS solution obtained superior kinematic characteristics, with significantly lower maximum values of velocity, acceleration, and jerk than those using B-spline and polynomial functions. Due to this, the dynamic forces and vibrations decreased. Moreover, the cam profile was designed considering the pressure angle, a, ensuring that it remained within acceptable bounds. However, the geometry parameters of the cam have not been mentioned for calculation in this research. Further research is essential, for the examination of the optimal geometry parameters of the cam profile for oscillating roller follower.

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APPENDIX

A. Polynomial Method

The polynomial, denoted by y, is expressed for the transfer function of the cam oscillating roller follower, as shown in (24). Taking the first, second, and third derivative of y, the velocity, acceleration, and jerk functions, are obtained:

$$y(\theta) = c_0 + c_1 \theta + c_2 \theta^2 + \dots + c_n \theta^n$$
 (24)

$$y'(\theta) = c_1 + 2.c_2\theta + 3.c_3\theta^2 ... + n.c_n\theta^{n-1}$$
 (25)

$$y''(\theta) = 2.c_2 + 3.2.c_3\theta ... + n(n-1)c_n\theta^{n-2}$$
 (26)

$$y'''(\theta) = 3.2.c_3 + \dots + n(n-1)(n-2)c_n\theta^{n-3}$$
 (27)

where, $c_0, c_1, c_2, ..., c_n$ are presented for the coefficients of the polynomial function, and n is the order of the polynomial.

B. B-spline Method

B-spline can be applied for the large boundary conditions. The transfer function and its kinematic characteristics of the cam oscillating roller follower using the B-spline function can be expressed as:

$$y(\theta) = \sum_{i=1}^{n} N_{i,p}(\theta) P_i$$
 (28)

$$y'(\theta) = \frac{dy(\theta)}{d\theta} = \sum_{i=1}^{n} N_{i,p}^{1}(\theta) P_{i}$$
 (29)

$$y''(\theta) = \frac{d^2 y(\theta)}{d\theta^2} = \sum_{i=1}^{n} N_{i,p}^2(\theta) P_i$$
 (30)

$$y'''(\theta) = \frac{d^3y(\theta)}{d\theta^3} = \sum_{i=1}^n N_{i,p}^3(\theta) P_i$$
 (31)

for
$$\theta \in [a, b], i = 1, 2, ..., n$$
.

In (28), $N_{i,p}(\theta)$ is the basis function. P_i is also presented for the control points of B-spline. $N_{i,p}^1, N_{i,p}^2, N_{i,p}^3$ are computed in (9).

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23216

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