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Model Reference Adaptive Controller for LTI Systems with Time-variant Delay

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Abstract—In this paper, a new Direct Model Reference Adaptive Control Procedure (DMRAC) for Linear Time-Invariant (LTI) delay systems is presented with the use of the concept of the command generator tracker which expands the class of processes that can now be controlled with zero output error. The stability of the error between the system and the model is guaranteed by the Lyapunov theory. The new algorithm is applied to control a perturbed delay system. Matlab simulation examples are given to demonstrate the usefulness of the algorithm.

Keywords—adaptive control; asymptotic stability; time delay systems; dynamical uncertainties

I. INTRODUCTION

The stability of time delay systems has been studied with the Lyapunov–Krasovskii and the Lyapunov–Razumikhin approach. These two concepts have been used in order to avoid the classical Lyapunov method. Authors in [1-3] give an overview of the stability of time delay systems with some advanced results. The rightmost roots of the characteristic are investigated in [4]. Authors in [5] studied the control of an MIMO nonlinear time delay system. Stability analysis and stabilization for Takagi–Sugeno (T–S) fuzzy systems with time delay have been studied in [6, 7]. In [8], a delay-dependent stabilization condition was proposed for the stability of a class T–S fuzzy time-delay system using homogeneous polynomials scheme and Polya's theorem with application on a truck-trailer model. Authors in [9] investigated the pre-specified performance for time-varying delays using model reduction, fuzzy logic, and LMI techniques. The PID controller has also been used in the stability of the time-delay systems [10]. The developed method guarantees gain and phase margins besides stability.

The introduction of adaptive control in uncertain time delay systems has been studied thoroughly. In [11], the author used the back stepping transformation where regulation was achieved despite the presence of partial measurements and disturbance. The adaptive identification of the parameters and the time delay of the time delay system were addressed in [12]. This identification is achieved with the use of the concept of transformation of the system in the parameterized form. The convergence of the identification error is guaranteed using the persistent excitation (PE) condition. Also, finite time convergence was assured using the terminal sliding mode. In [13], the author applied a sliding mode controller to stabilize uncertain time-delay chaotic systems. The proposed controller was robust against time-delays, parameter uncertainties and disturbances. The H-infinity theory has also been used to control time-delay systems. In [14], time-delays appeared in the network used in the feedback loop. The delay-dependent stability criterion was derived from the Lyapunov - Krasovskii theorem and the Linear Matrix Inequality (LMI). The H2, H-infinity and the LMI concepts have been used for discrete time delay uncertain systems. Authors in [15] used the past values of the states and the outputs, and were able to stabilize the system with time-varying delays. Finite time stability of time-delay systems has been investigated in [17-18] with the utilization of the homogeneity theorem. The observer design of time delay systems was used in [19] for a switched singular system where two design methods were used and in [20], a Luenberger-like observer has been used to estimate the unknown inputs for a large class of linear systems. The output regulation of time-delay systems has also been investigated in [21] by using the adaptive concept and the observer design using RBF neural network systems to approximate unknown functions. In [22] the well known Lyapunov–Krasovskii theorem was used to investigate the output stabilization for time-delay nonholonomic systems.

The simple MRAC of MIMO plants was first proposed in [23]. This class of algorithms does not require full state access or satisfaction of perfect model conditions. Asymptotic stability is ensured provided that the plant is Almost Strictly Positive Real (ASPR). Authors in [24] extended the original algorithm to a class of plants which violates this condition. This approach involved designing a supplementary feedforward filter to be included in parallel with the original plant resulting in a new augmented plant which had to satisfy the same strictly positive real condition. Unfortunately, the tracking error was not the true difference between the plant and the model outputs since it included the contribution of the supplementary feedforward filter which led to an asymptotically stable error [25-28]. The application of adaptive fuzzy control can be found in [33, 34]. The authors considered the internal model for controlling DC-DC converters. Adaptive control is also used in many industrial fields, while authors in [35] used it for controlling UAV systems.
Authors in [36] developed a saturated command for planar systems where the stabilization is achieved in finite time using just a simple proportional derivative corrector PD whose parameters are optimally adapted. This finite time stability is analyzed with Lyapunov’s theory and homogeneity concept. Author in [37] aimed to replace a mechanical cam system with an electromagnetic actuator, the PD corrector in the presence of noise at high frequencies by which replaces the classic mechanical valve actuator, the PD technique improves the efficiency of the mechanical engine and adaptively according to the estimated speed. This adaptive measurement without a speed sensor. The position is deducted from the explosion gas and therefore the combustion engine rotation. Electromagnetic force is generated by making a velocity movement allowing the admission and exhaust of the move linearly allowing the admission and exhaust of the

II. DIRECT MODEL REFERENCE ADAPTIVE CONTROL

The model reference adaptive control is considered for the non-linear plant:

\[
x_p(t) = A_p x_p(t) + A_n x_n(t - \tau(t)) + B_p u_p(t) + f(x_p) \quad y_p(t) = C_p x_p(t)
\]

where \( x_p(t) \) is the \( (n\times1) \) state vector, \( u_p(t) \) is the \( (m\times1) \) control vector, \( y_p(t) \) is the \( (q\times1) \) plant output vector, \( f(t) \) is an \( (n\times1) \) vector of nonlinearities, \( A_n, B_p \) are matrices with appropriate dimensions, and \( \tau(t) \) is the time delay that verifies assumption (2) as stated below. We assume that the parameters of the linear part of the plant model are uncertain, i.e. only known within certain finite bounds. The range of the plant parameters is assumed to be known and bounded with:

\[
a_{ij} \leq a_p(i, j) \leq b_p, i, j = 1, \ldots, n
\]

\[
b_{ij} \leq b_p(i, j) \leq b_p, i, j = 1, \ldots, n
\]

• Assumption 1:

The non-linear function \( f(x) \) is Lipschitz in its arguments, that means \( |f(x_1) - f(x_2)| < L|x_1 - x_2| \) where \( L>0 \) is the constant of Lipschitz, \( |x| \) is the Euclidean norm and \( x_1, x_2 \) belong to a compact set \( \Omega \subset R^n \).

• Assumption 2:

The derivative of the delay system \( \tau(t) \) verifies:

\[
\frac{d\tau(t)}{dt} \leq \tau_i
\]

The objective of this paper is to find, without explicit knowledge of \( A_p, B_p \) and for non-linear\( f(x_p) \), the control \( u_p(t) \) such that the plant output vector \( y_p(t) \) follows the reference model given by:

\[
x_n(t) = A_n x_n(t) + A_n x_n(t - \tau) + B_n u_n(t)
\]

\[
y_n(t) = C_n x_n(t)
\]

The output \( y_n \) is the desired response to the set point command \( u_n \). The model incorporates the desired behavior of the plant, but its choice is not restricted. In particular, the order of the plant may be much larger than the order of the reference model. The ideal control law that generates perfect output tracking and ideal state trajectories is assumed to be a linear combination of the model states and the model input (see [29]). In our case, we suppose that the ideal state, its delay and the ideal input are related to the model state. Its delay and the model input are related by:

\[
\begin{bmatrix}
x_p^*(t) \\
u_p^*(t)
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
x_n(t) \\
u_n(t)
\end{bmatrix}
\]

The perfect output tracking means that the ideal output \( y^*(t) \) is equal to the output model \( y_n(t) \) which means:

\[
y_n(t) = C_p x_n(t) + C_p S_{11} x_n(t) + C_p S_{12} u_n(t)
\]

\[
y_n(t) = C_n x_n(t)
\]

Taking into account (5.0) that the ideal state \( x^*(t) \) verifies this relation and the assumption that the command \( u_n(t) \) is constant (in the case where the input is not a constant, we can always find a dynamic system to generate \( u_n(t) \) with a constant input), the derivative of \( x^*(t) \) can be written as:

\[
\frac{dx^*_p(t)}{dt} = \frac{d}{dt}(S_{11} x_n(t) + S_{12} u_n(t))
\]

\[
= S_{11}(A_n x_n(t) + A_n x_n(t - \tau) + B_n u_n(t) + B_n u_n(t - \tau)))
\]

\[
= S_{11}(A_n x_n(t) + S_{12} A_n x_n(t - \tau) + S_{12} B_n u_n(t) + S_{12} B_n u_n(t - \tau))
\]

\[
= A_p x_p(t) + A_p x_p(t - \tau) + B_p u_p(t)
\]

\[
= A_n(S_{11} x_n(t) + S_{12} u_n(t)) + A_n(S_{11} x_n(t - \tau) + S_{12} u_n(t - \tau))
\]

\[
+ S_{12} B_n u_n(t) + B_p(S_{31} x_n(t) + S_{32} u_n(t))
\]

\[
\frac{dx^*_p(t)}{dt} = \frac{d}{dt}(S_{11} x_n(t) + S_{12} u_n(t))
\]

Using (5.1)-(5.2), we obtain the following algebraic system:

\[
\begin{bmatrix}
S_{11} A_p &=& A_p S_{11} + B_p S_{21} \\
S_{12} B_n &=& A_p S_{12} + B_p S_{22} \\
S_{11} A_n &=& A_n S_{11} \\
S_{12} B_n &=& A_n S_{12} \\
C_p S_{11} &=& C_n \\
C_p S_{12} &=& 0
\end{bmatrix}
\]

which can be written as:
In the system (6) we have more unknowns than equations, so the solution almost always exists. When \( A_p, \) \( A_{mp}, \) and \( B_{mr} \) are null, then the system and the model are without delay and we get the equations given in [29]. The adaptive control law based on the extended Command Generator Tracker (CGT) approach is given by:

\[
u_p(t) = K_r(t)e_p(t) + K_r(t)x_p(t) + K_x(t)u_m(t)
\]

(7)

The adaptive law (7) has been applied for linear systems [30, 31]. Here we aim to extend it to a linear time delay system described by (1) by adding a delay in the input and output in the model (4). The tracking error is given by:

\[
e_p(t) = y_p(t) - y_p(t)
\]

(8)

Defining the vector \( r(t)(n \times 1) \) as:

\[
r(t) = \left[ \begin{array}{c} (y_p(t) - y_p(t))^T \ x_p(t) \ u_m(t) \end{array} \right]^T
\]

(9)

the control \( u_p(t) \) is written in a compact form as:

\[
u_p(t) = K(t)r(t)
\]

(10)

where

\[
K(t) = K_p(t) + K_r(t)
\]

(11)

\[
K_p(t) = \left[ \begin{array}{c} y_p(t) - y_p(t) \end{array} \right] r(t) T_p^p, \ T_p^p \geq 0
\]

(12)

\[
K_r(t) = \left[ \begin{array}{c} y_p(t) - y_p(t) \end{array} \right] r(t) T_r^r, \ T_r^r > 0
\]

(13)

III. STABILITY STUDY

The first step of the demonstration is to design a positive definite quadratic form in the state variables \( e_p(t) \) and \( K_r(t) \) of the adaptive system. \( T^{-1} \) is assumed to be a symmetric positive definite matrix. Then an appropriate choice of the Lyapunov-Krasovskii functional [32] is:

\[
V = e_p^T Pe_p + \int_0^T e_p^T(\alpha)Qe_p(\alpha)d\alpha + T r \left[ S(K - K)T^{-1}(K - K) S^T \right]
\]

(14)

where \( Tr \) is the trace of a matrix. Its time derivative is:

\[
\dot{V} = e_p^T P e_p + e_p^T P e_p + e_p^T(t)Qe_p(t) - (1 - \tau) e_p^T(t - \tau)Qe_p(t - \tau) + 2T r \left[ S(K - K)T^{-1} K S^T \right]
\]

(15)

where \( P, Q \) are symmetric positive definite matrices of size \( n \times n \), \( K \) is a \( m \times n \) matrix and \( S \) is a non-singular \( m \times m \) matrix.

Since the matrix \( K \) appears only in the function \( V \) and not in the control algorithm, it is called fictitious gain matrix. It has the same dimension as \( K \) where:

\[
\dot{K} = K_r C_p e_p + \dot{K} x_p + K_x u_m
\]

(16)

The four gains \( K_r, \dot{K}, K_x \) and \( K_x \) are as \( K \) fictitious.

Then we take the equation of the error using the fact that \( e_p = x_p - x_p \) to find:

\[
e_p = A_p x_p + A_p x_p(t - \tau) + B_p u_p + f(x_p) - A_p x_p(t - \tau) - B_p u_p - f(x_p)
\]

(17)

\[
= A_p x_p(t - \tau) - B_p u_p - f(x_p)
\]

(17)

Also:

\[
e_p = A_p x_p + A_p x_p(t - \tau) + B_p u_p - u_p + f(x_p) - f(x_p)
\]

If we set: \( df = f(x_p) - f(x_p) \) and substitute \( u_p \) from (5.0) and \( u_p \) from (7), we get:

\[
\dot{e}_p = A_p e_p + A_p e_p(t - \tau) + B_p \left[ S_{21} x_m + S_{22} u_m - K_r x_m - K_r u_m - K_r C_p e_p \right] + df
\]

(18.a)

\[
= A_p e_p + A_p e_p(t - \tau) + B_p \left[ S_{21} x_m + S_{22} u_m - K_r r - C_p e_p T_r r \right] + df
\]

(18.b)

Then the adaptive system is described by:

\[
\dot{e}_p = A_p e_p + A_p e_p(t - \tau) + B_p \left[ S_{21} x_m + S_{22} u_m - K_r r - C_p e_p T_r r \right] + df
\]

(19)

Substituting (19) and (20) in (15), we get:

\[
\begin{align*}
V &= A_p e_p + A_p e_p(t - \tau) + B_p \left( S_{21} x_m + S_{22} u_m - K_r r \right)^T - e_p^T P \left( -C_p e_p T_r r \right) + e_p^T P \left( -C_p e_p T_r r \right)^T + 2T r \left[ S(K - K)T^{-1}(C_p e_p T_r r)^T S^T \right] + e_p^T(t) Q e_p(t) - (1 - \tau) e_p^T(t - \tau) Q e_p(t - \tau) + df
\end{align*}
\]

(21)

We can write it as:
\[
V = e^T(t) (PA_\tau + A^T_\tau P + Q)e_\tau + e^T(t - \tau) A^T_\tau Pe_\tau \\
+ 2e^T(t) PB_\tau (S_{12}x_m + S_{22}u_m) - 2e^T(t) C^T_\tau S^T S K_\tau r \\
-2e^T(t) C^T_\tau S^T S K_\tau r - (1 - \tau) e^T(t - \tau) Qe_\tau (t - \tau) + df
\]

Knowing that for two vectors \(U(1,1)\) and \(V(1,1)\) have:

\[
Tr[U^TV] = VU
\]

therefore:

\[
V = e^T(t) (PA_\tau + A^T_\tau P + Q)e_\tau + e^T(t - \tau) A^T_\tau Pe_\tau \\
+ 2e^T(t) PB_\tau (S_{12}x_m + S_{22}u_m) - 2e^T(t) C^T_\tau S^T S K_\tau r \\
-2e^T(t) C^T_\tau S^T S K_\tau r - (1 - \tau) e^T(t - \tau) Qe_\tau (t - \tau) + df
\]

which means that:

\[
V = e^T(t) (PA_\tau + A^T_\tau P + Q)e_\tau + e^T(t - \tau) A^T_\tau Pe_\tau \\
+ 2e^T(t) PB_\tau (S_{12}x_m + S_{22}u_m) - 2e^T(t) C^T_\tau S^T S K_\tau r \\
-2e^T(t) C^T_\tau S^T S K_\tau r - (1 - \tau) e^T(t - \tau) Qe_\tau (t - \tau) + df
\]

Substituting \(K_\tau = K, C_\tau e_\tau, x_m, \) and \(K_\tau u_m\) in (24) we get:

\[
V = e^T(t) \left[ P(A_\tau - B_\tau \bar{K}, C_\tau) + (A_\tau - B_\tau \bar{K} C_\tau)^T P + Q \right] e_\tau \\
+ e^T(t - \tau) A^T_\tau Pe_\tau + 2e^T(t) PB_\tau (S_{12}x_m + S_{22}u_m) \\
-2e^T(t) C^T_\tau S^T S K_\tau r - (1 - \tau) e^T(t - \tau) Qe_\tau (t - \tau) + df
\]

Thus, if we set:

\[
\begin{bmatrix}
(S_{21} - \bar{K}_s)x_m + (S_{22} - \bar{K}_s)u_m
\end{bmatrix} = 0
\]

or \(\bar{K}_s = S_{21}\) and \(\bar{K}_s = S_{22}\) (none of which is required for implementation), the derivative of \(V\) becomes:

\[
V = e^T(t) \left[ P(A_\tau - B_\tau \bar{K}, C_\tau) + (A_\tau - B_\tau \bar{K} C_\tau)^T P + Q \right] e_\tau \\
+ e^T(t - \tau) A^T_\tau Pe_\tau + 2e^T(t) PB_\tau (S_{12}x_m + S_{22}u_m) \\
-2e^T(t) C^T_\tau S^T S K_\tau r - (1 - \tau) e^T(t - \tau) Qe_\tau (t - \tau) + df
\]

Taking into account the Assumption 2, the derivative of the Lyapunov function verifies (27):

\[
V \leq \begin{bmatrix}
e_{\tau}(t) \\
e_{\tau}(t - \tau)
\end{bmatrix} \begin{bmatrix}
Q_{1} & PA_\tau \\
A^T_\tau P & -(1 - \tau)Q
\end{bmatrix} \begin{bmatrix}
e_{\tau}(t) \\
e_{\tau}(t - \tau)
\end{bmatrix} + df
\]

with

\[
Q_{1} = P(A_\tau - B_\tau \bar{K}, C_\tau) + (A_\tau - B_\tau \bar{K} C_\tau)^T P + Q
\]

From (12), \(T_\tau\) is positive semi-definite, so (27) becomes:

\[
V \leq \begin{bmatrix}
e_{\tau}(t) \\
e_{\tau}(t - \tau)
\end{bmatrix} \begin{bmatrix}
Q_{1} & PA_\tau \\
A^T_\tau P & -(1 - \tau)Q
\end{bmatrix} \begin{bmatrix}
e_{\tau}(t) \\
e_{\tau}(t - \tau)
\end{bmatrix} + df
\]

Let’s take:

\[
E_e = \begin{bmatrix}
e_{\tau}(t) \\
e_{\tau}(t - \tau)
\end{bmatrix} \quad \text{and} \quad Q_2 = \begin{bmatrix} Q_{1} & PA_\tau \\
A^T_\tau P & -(1 - \tau)Q \end{bmatrix}
\]

Then (29) is rewritten as:

\[
V \leq -E_e^T Q_2 E_e + df
\]

When \(df\) is equal to zero, the error is asymptotically stable if and only if \(Q_2 = Q_2^T\) is positive semi-definite. In the case where \(df\) is different from zero and satisfies the Assumption 1, then the derivative of the Lyapunov function verifies:

\[
V \leq -E_e^T Q_2 E_e + |df| \leq -E_e^T Q_2 E_e + L |x - x^*| = -E_e^T Q E_e + L |e_\tau| \\
\leq -\lambda_{\min}(Q_2) |E_e|^T E_e + L |e_\tau| \leq -\lambda_{\min}(Q_2) |E_e|^T L |e_\tau| \leq 0
\]

where \(\lambda_{\min}(Q_2)\) stands for the lowest eigenvalue of \(Q_2\), which is a positive number since \(Q_2 = Q_2^T \geq 0\). The last inequality implies that the error \(e_\tau\) is ultimately uniformly stable, which means that it belongs to a compact set around the origin. This set can be rendered much lower if we select \(\lambda_{\min}(Q_2)\) to be...
Theorem:

The control given by (10), and the adaptive laws given by (11), (12) and (13) of the non-linear uncertain system (1) that verifies the Assumption 1 lead to an asymptotically stable error between the system and the model if and only if there are two $P$, $Q$ matrices $P = P^T > 0$ and $Q = Q^T ≥ 0$ such that:

1) The matrix

$$Q_2 = \begin{bmatrix} H & PA \\ A^TP & -(1-\tau_p)Q \end{bmatrix}$$

where

$$H = P(A_p-B_p\tilde{K}_p, C_p)+(A_p-B_p\tilde{K}_p, C_p)^TP+Q$$

is positive semi-definite for some matrix $\tilde{K}_p$.

2) $P\bar{B}_p = (GC)^T$, $G = (S^TS)^{-1}$, for a non-singular matrix $S$.

3) $[f(x_1) - f(x_2)] < L|x_1 - x_2|$, $L > 0$, $x_1, x_2 \in R^3$

$$\left[ P(A_p-B_p\tilde{K}_p, C_p)+(A_p-B_p\tilde{K}_p, C_p)^TP \right] = -Q$$

$$= -\alpha I$$

$$PB_p = (GC)^T \quad \tau_p ≥ 0$$

where $\alpha \in R^+$, and $I$ is the identity matrix.

These relations imply that the feedback system is SPR for large $\alpha$, and so that the original linear system is ASPR.

IV. Simulation

During simulations, it is required that the output of the system tracks the output reference. The controlled system is given by:

$$x_p(t) = A_p x_p(t) + A_1 x_p(t - \tau(t)) + B_p u_p(t)$$

$$y_p(t) = C_p x_p(t)$$

with $A_p = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$, $A_1 = \begin{bmatrix} 5 & 6 \\ 6 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 6 \end{bmatrix}$.

The transfer function of the reference model is given by:

$$G_n(s) = \frac{2}{s+1}$$
Fig. 2. Outputs of the system and the model without perturbation

Fig. 3. The system command $u(t)$

Fig. 4. The gains $K_e$, $K_x$, and $K_u$

Fig. 5. Outputs of the system and the model with perturbation $T_e = T_i = 1 \times I_{13}$

Fig. 6. The system command $u(t)$

Fig. 7. Outputs of the system and the model with perturbation $T_e = T_i = 10 \times I_{13}$
C. Case 3: With Perturbation and $T_p = T_i = 10 \times I_{3,3}$

In this case and in order to overcome the drawback that has appeared in the previous case, we augmented the adjustable parameter $T_p$, $T_i$ as $T_p = T_i = 10 \times I_{3,3}$. Figure 7 shows a perfect tracking compared to Figure 5 and Figure 8 shows the effect of the controller in overcoming the perturbation and letting the system output track the reference model. Note that this command is bounded and does not represent a high oscillation. Figure 9 presents the gains that are bounded and they are adjusted to construct the system input.

Fig. 8. The system command $u(t)$

Fig. 9. The gains $K_e$, $K_e$, and $K_i$

V. CONCLUSION

This paper presents an adaptive command applied for a perturbed time delay system. The Lyapunov’s theory has been addressed in order to achieve a robust command against the uncertainty which is inherent in all real systems. The simulation results confirm the robustness of the developed command.


